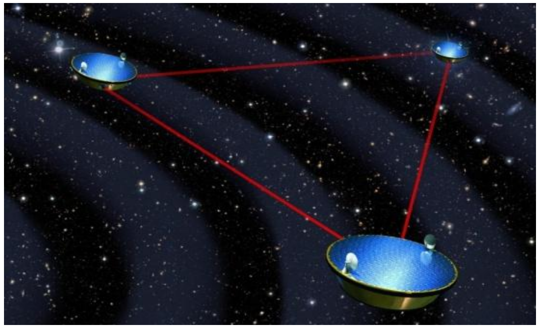
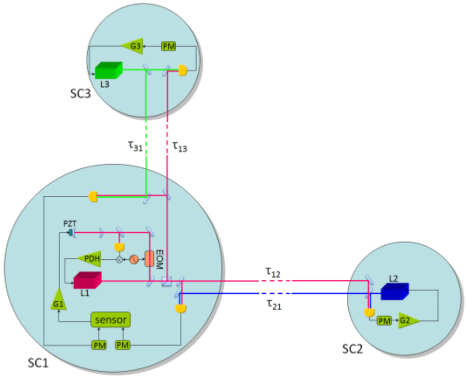
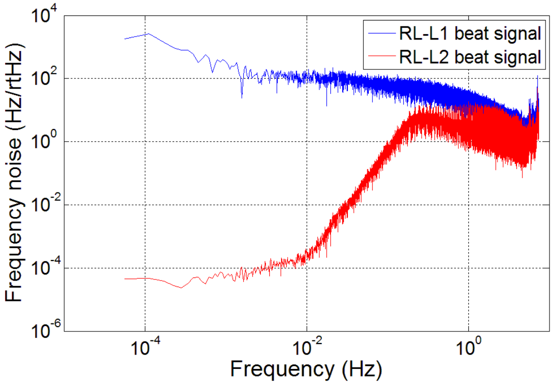
Arm Locking for Laser Interferometer Space Antenna







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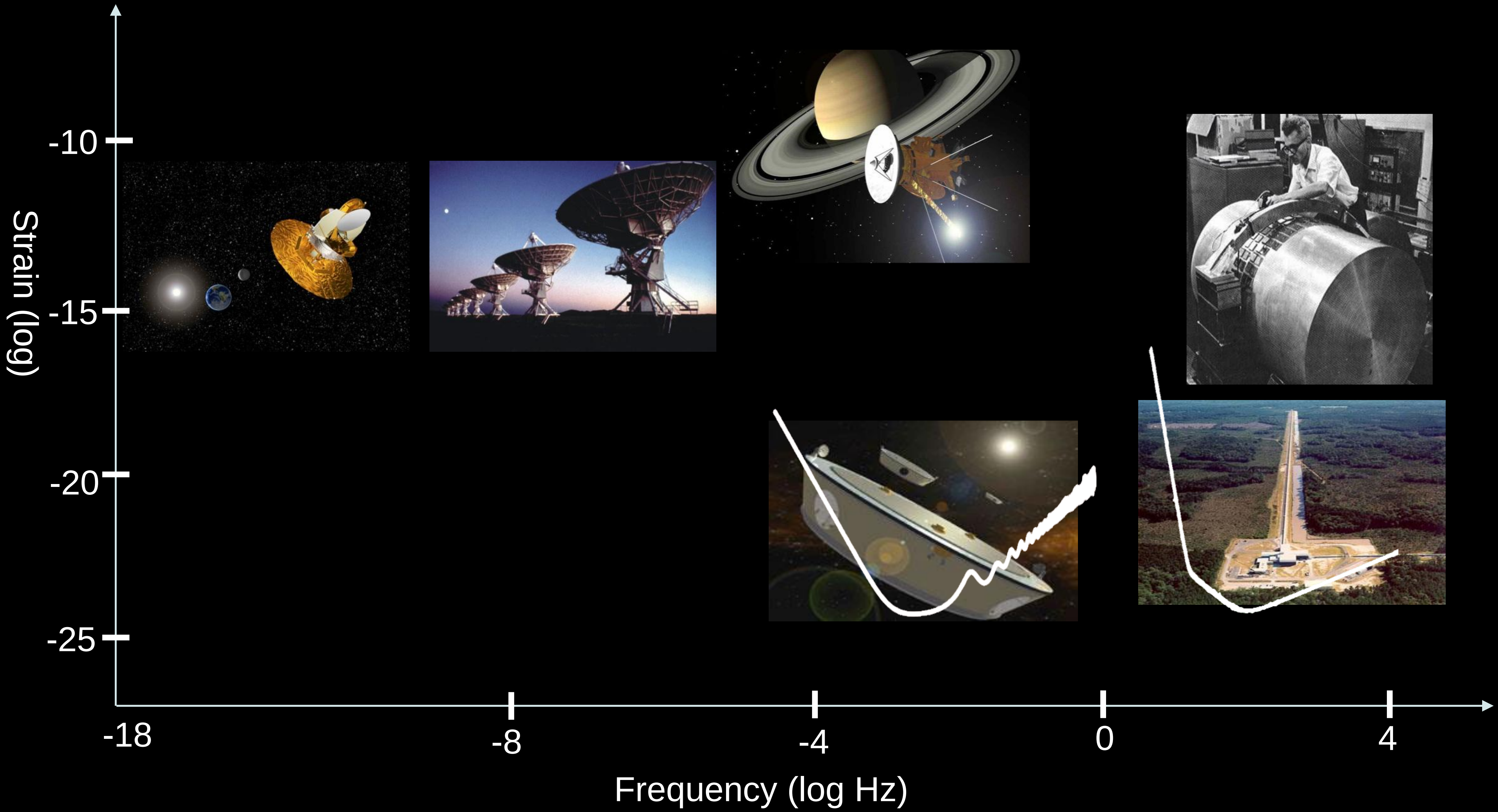
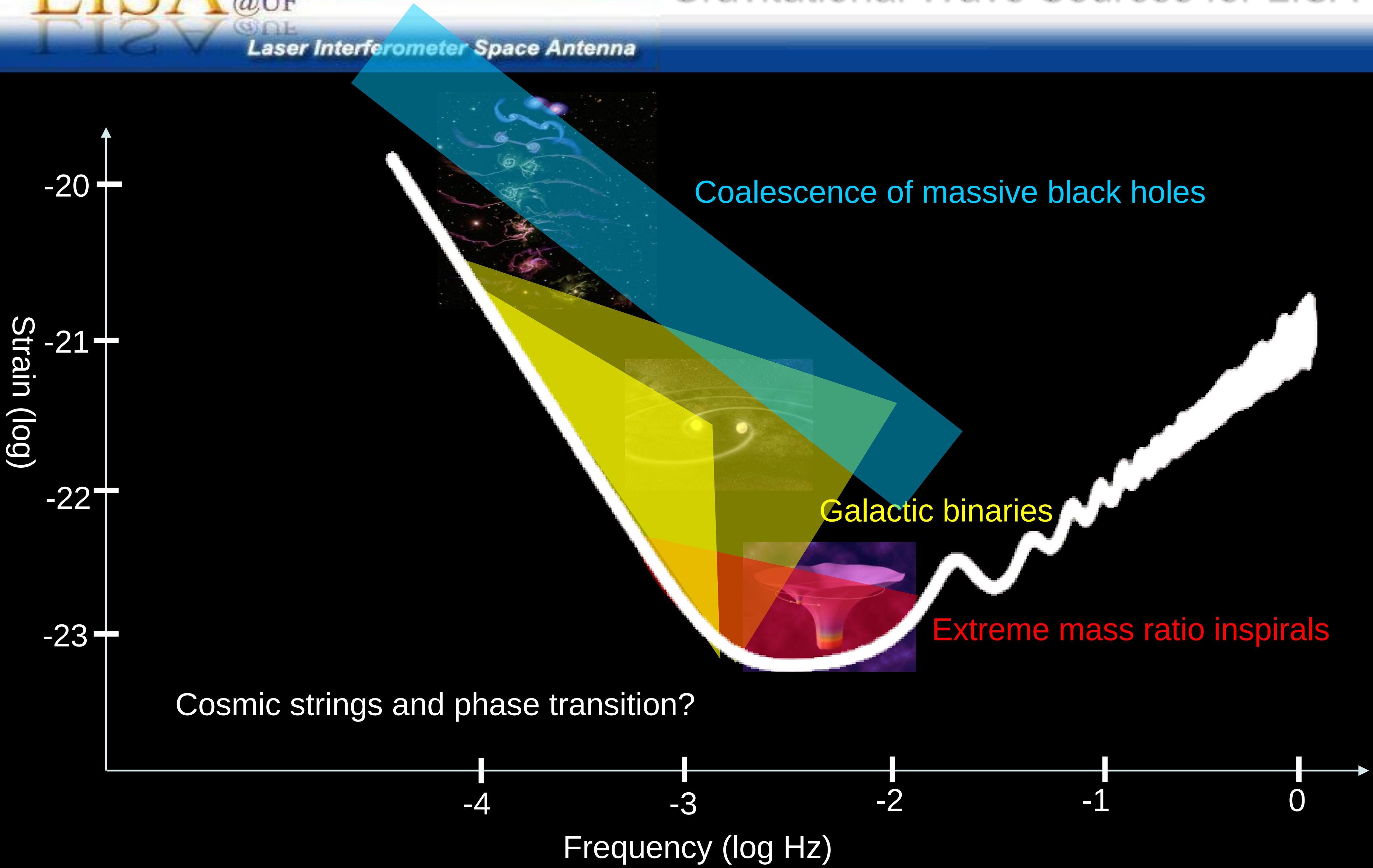
Gravitational Wave Astronomy

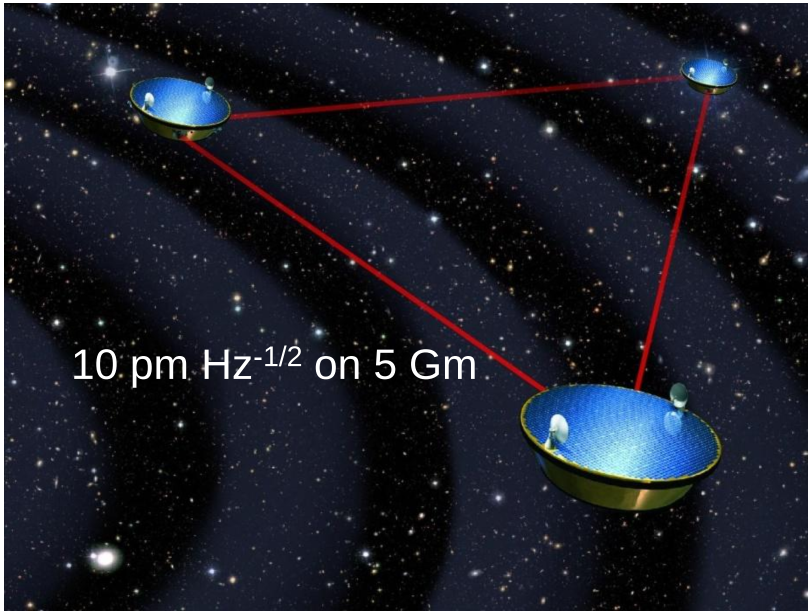


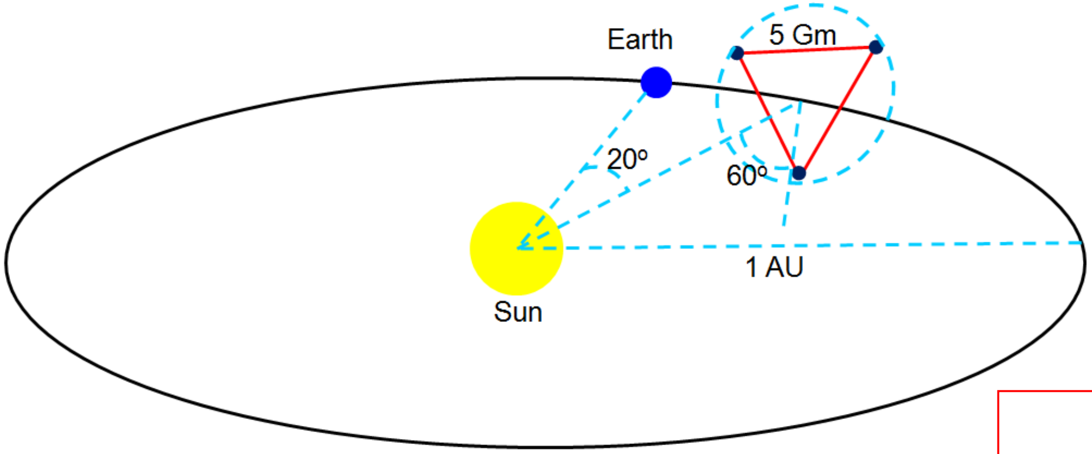
Gravitational Wave Sources for LISA



LISA Concept

- Detect and observe GWs from 0.03 mHz to 1 Hz
- Three spacecraft in independent heliocentric orbits
- Large separation 5 × 10^9 m, variation ±1%
- Trailing Earth by 20 degrees, out of the ecliptic by 60 degrees





Key Technologies

- picometer precision interferometry in space
- drag-free proof masses (to be tested by LPF)

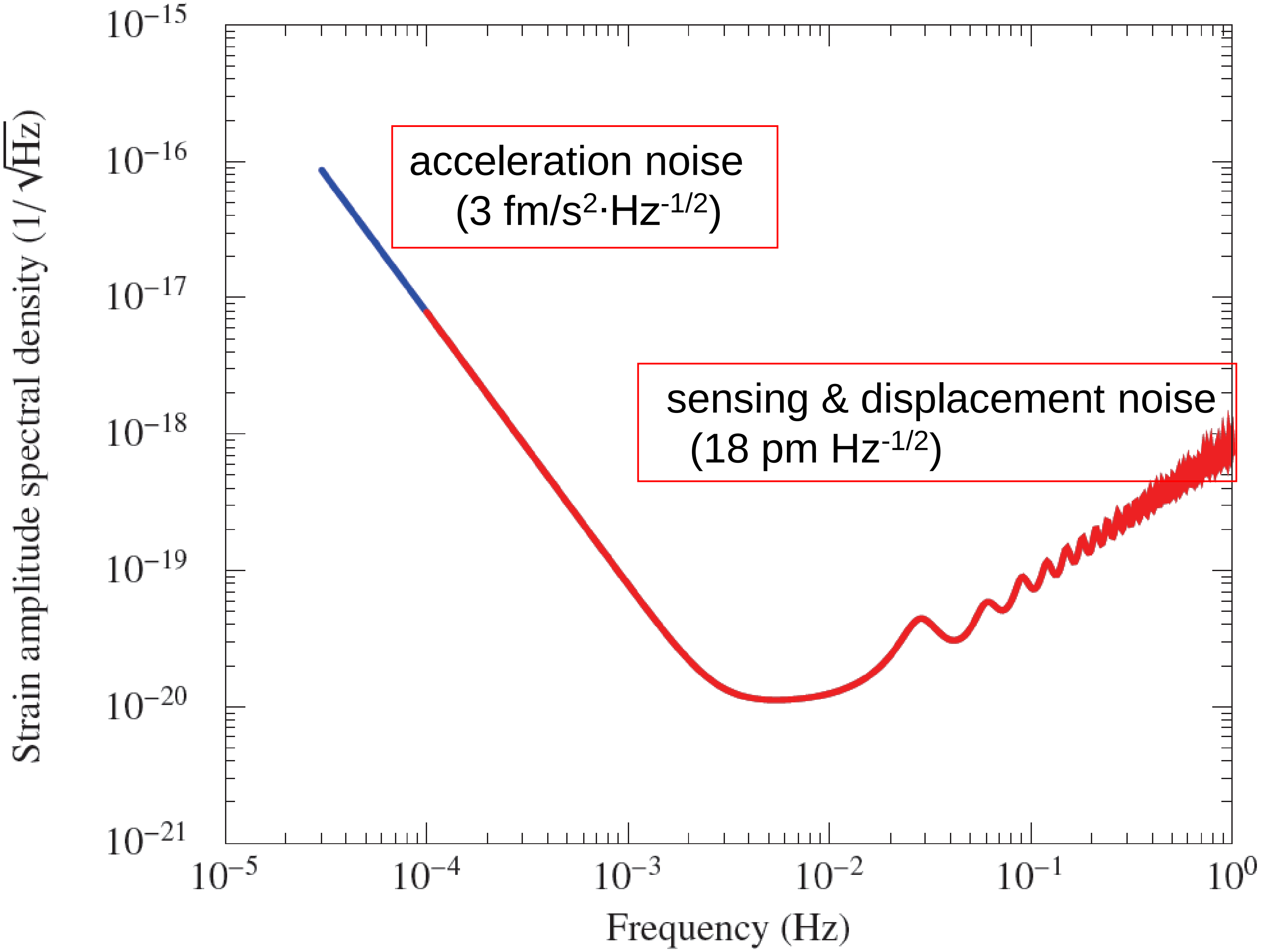
Strain Sensitivity

Acceleration noise:

- Electrostatic force
- Thermal effect
- Magnetic field
- Gravity gradient
- …

Sensing & displacement noise:

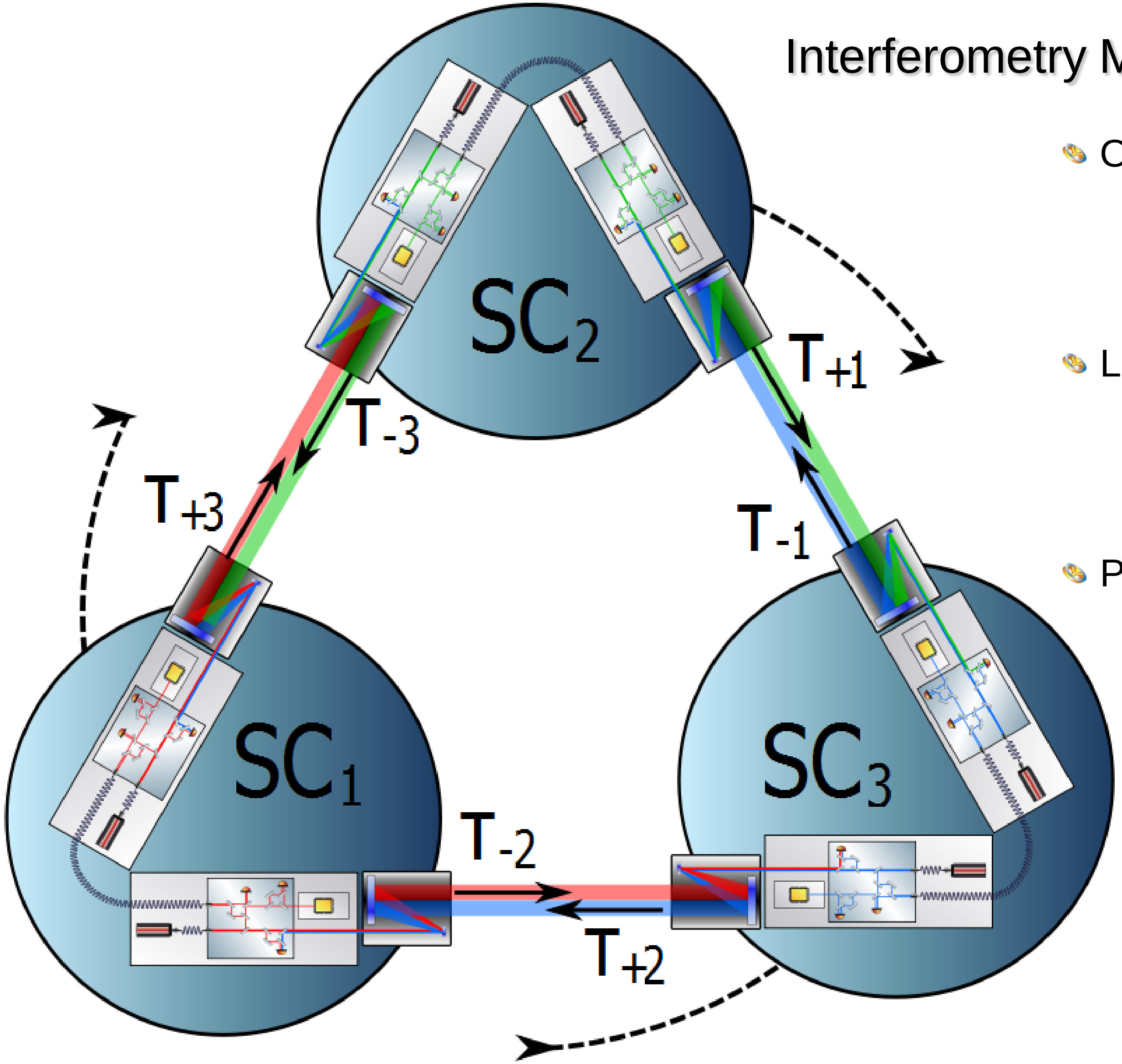
- Shot noise
- Pathlength noise
- Laser frequency noise
- Clock noise
- …



acceleration noise ($3\ \mathrm{fm/s^2 \cdot Hz^{-1/2}}$)

sensing & displacement noise ($18\ \mathrm{pm\ Hz^{-1/2}}$)

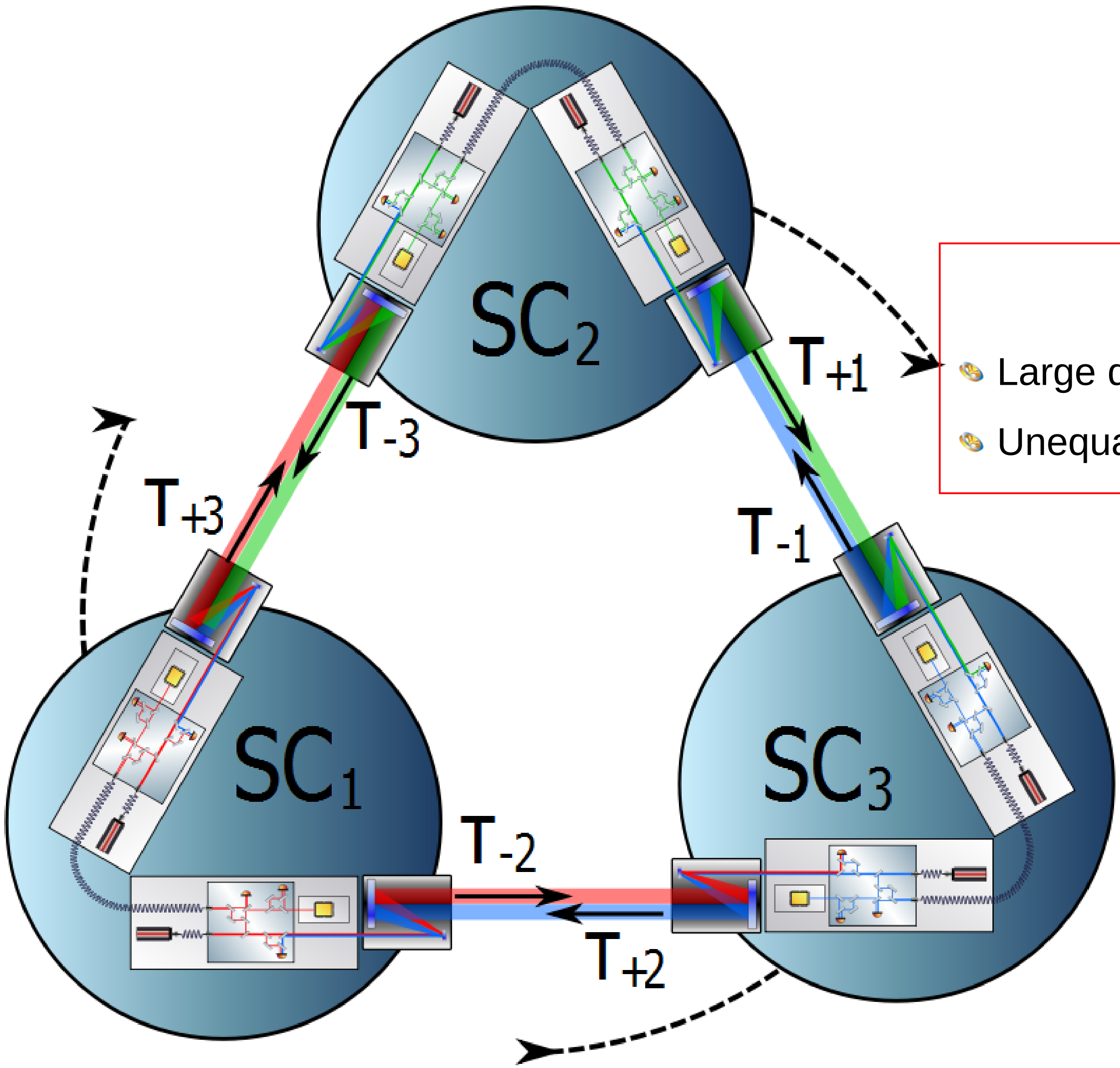
Architecture of IMS

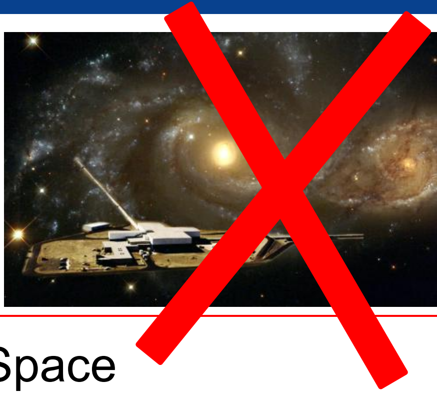


Interferometry Measurement System (IMS)

- Optical system
  - optical bench
  - telescope
- Laser system
  - 1064 nm Nd:YAG or laser diode
  - frequency stabilization subsystem
- Phase measurement system
  - phasemeter

Architecture of IMS



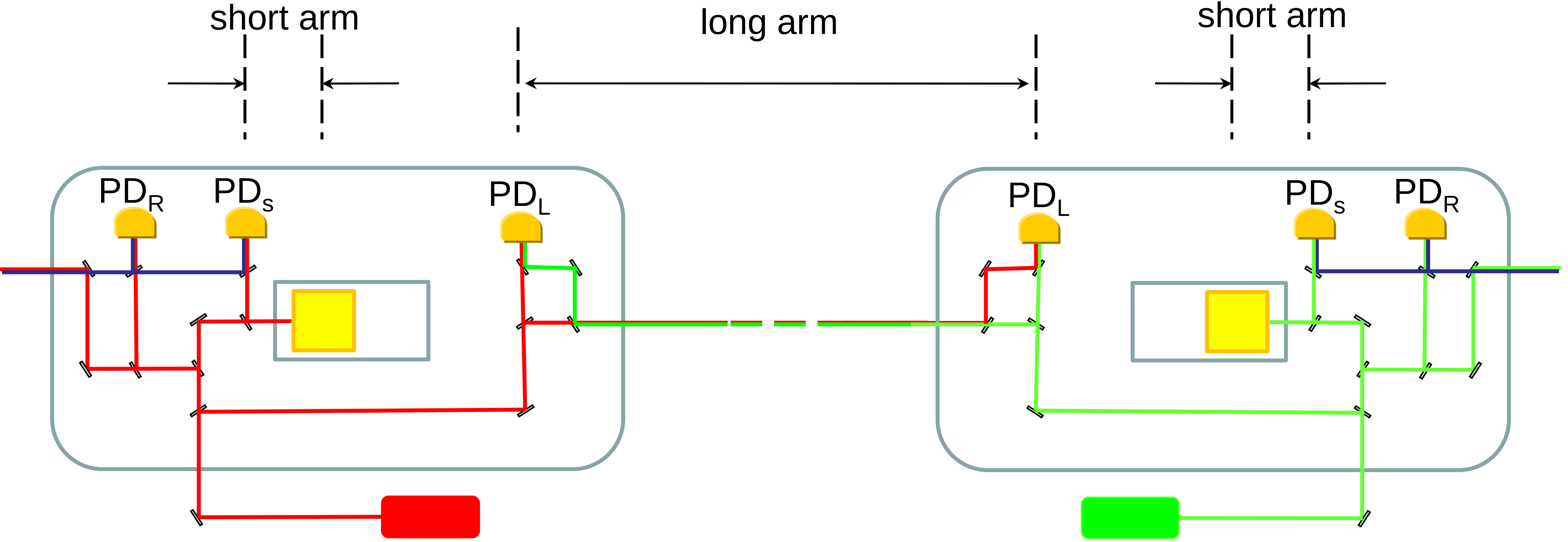


LISA ≠ LIGO In Space

- Large distance and diffraction – optical transponder
- Unequal and time-varying arm lengths

- Heterodyne interferometry in MHz
- Individual phase measurement with respect to the local clock
- Synthesize Michelson or Sagnac interferometer in post-processing

From Proof Mass to Proof Mass



- Proof mass relative to the optical bench:

  Reference interferometry (PD_R) $\varphi_R(t) = \Phi_l(t) - \Phi_a(t) + N_a(t)$

  Short arm interferometry (PD_S) $\varphi_S(t) = \Phi_l(t) - \Phi_a(t) + \frac{2\pi}{\lambda}\Delta L_{pm}(t) + N_a(t)$

- Optical bench to optical bench

  Long arm interferometry (PD_L) $\varphi_L(t) = \Phi_l(t) - \Phi_f(t - \tau_{ji}(t)) + N_{Trans}(t - \tau_{ji}(t)) + h_{ji}(t)$

Laser Frequency Control

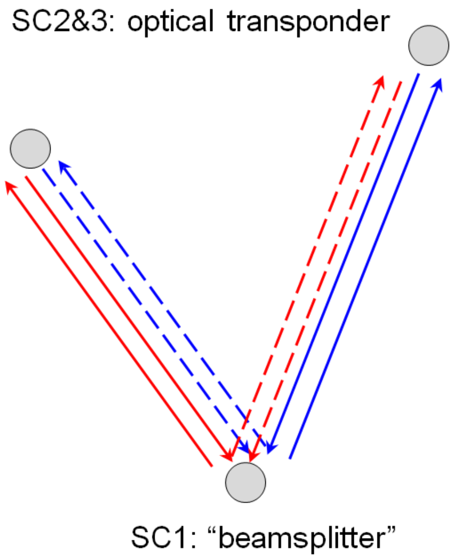
- Noise allocation for laser frequency noise: 2 pm $\mathrm{Hz}^{-1/2}$

Converted into phase noise: $\delta\varphi = \frac{2\pi}{\lambda}\delta L = 1.2 \times 10^{-5}\ \mathrm{rads\ Hz}^{-1/2}$

Converted into frequency noise: $\delta\nu = \frac{\delta L}{\Delta L}\nu = 1.1 \times 10^{-6}\ \mathrm{Hz\ Hz}^{-1/2}$

- Free-running laser frequency noise: (10 kHz/f) Hz $\mathrm{Hz}^{-1/2}$

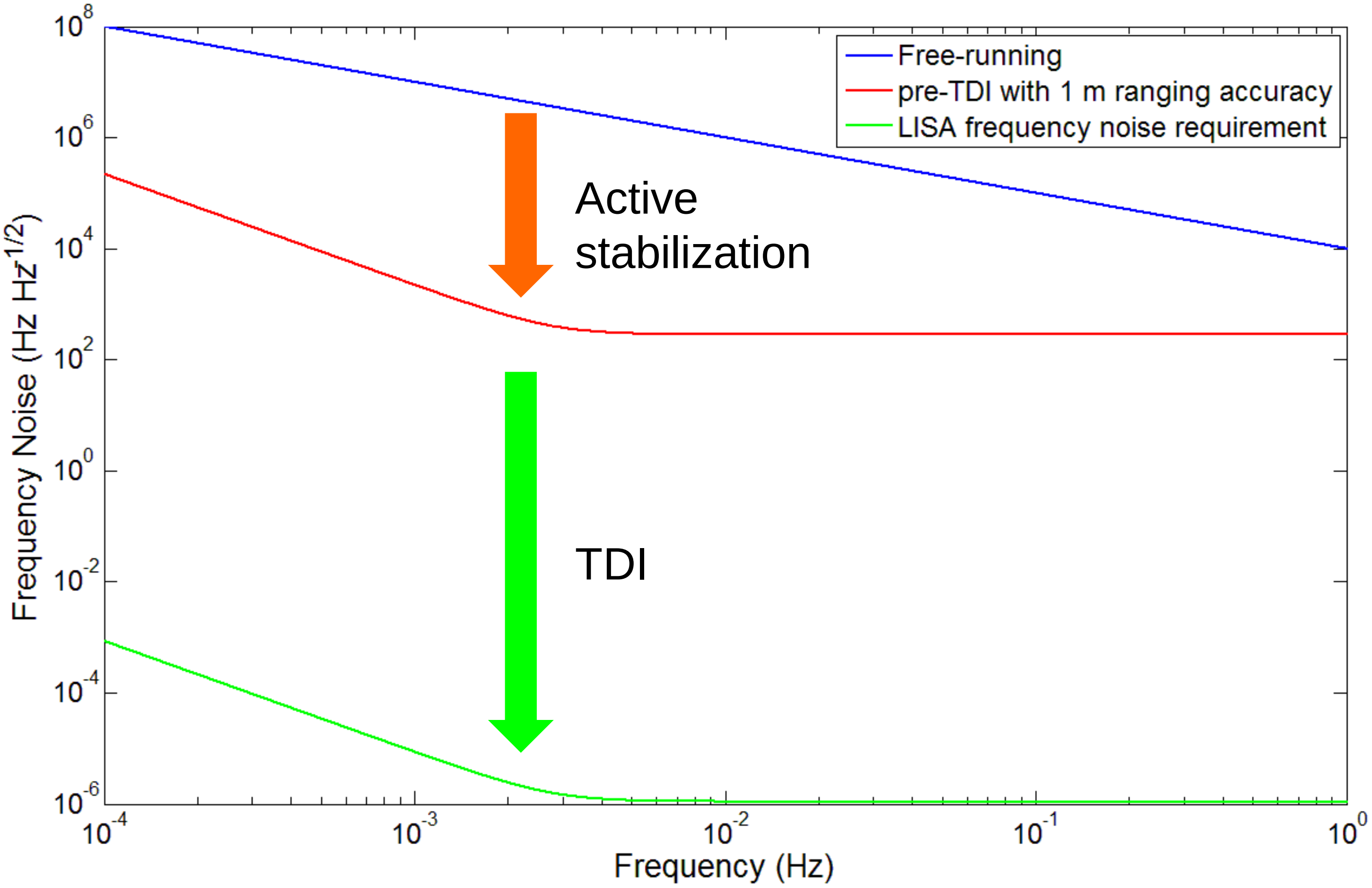
- How can we suppress the laser frequency noise by more than 12 orders of magnitude?



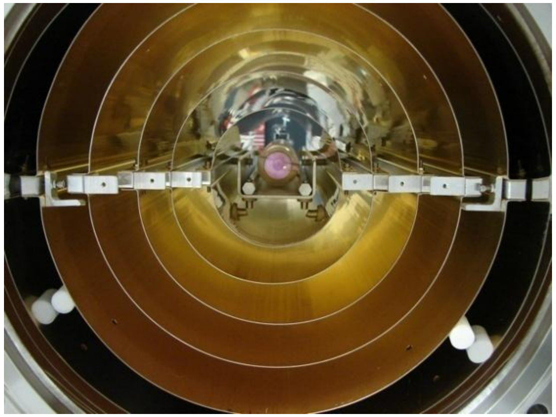
- Time Delay Interferometry
  Equal-arm Michelson interferometer in post-processing
- Require arm length knowledge
  Limited by 1 m ranging accuracy (if PRN ranging is used)

$$\delta\nu_{\mathrm{pre-TDI}}(f) < 282 \times \sqrt{1 + \left(\frac{2.8\ \mathrm{mHz}}{f}\right)^4}\ \mathrm{Hz\ Hz}^{-1/2}$$

Active Laser Frequency Stabilization

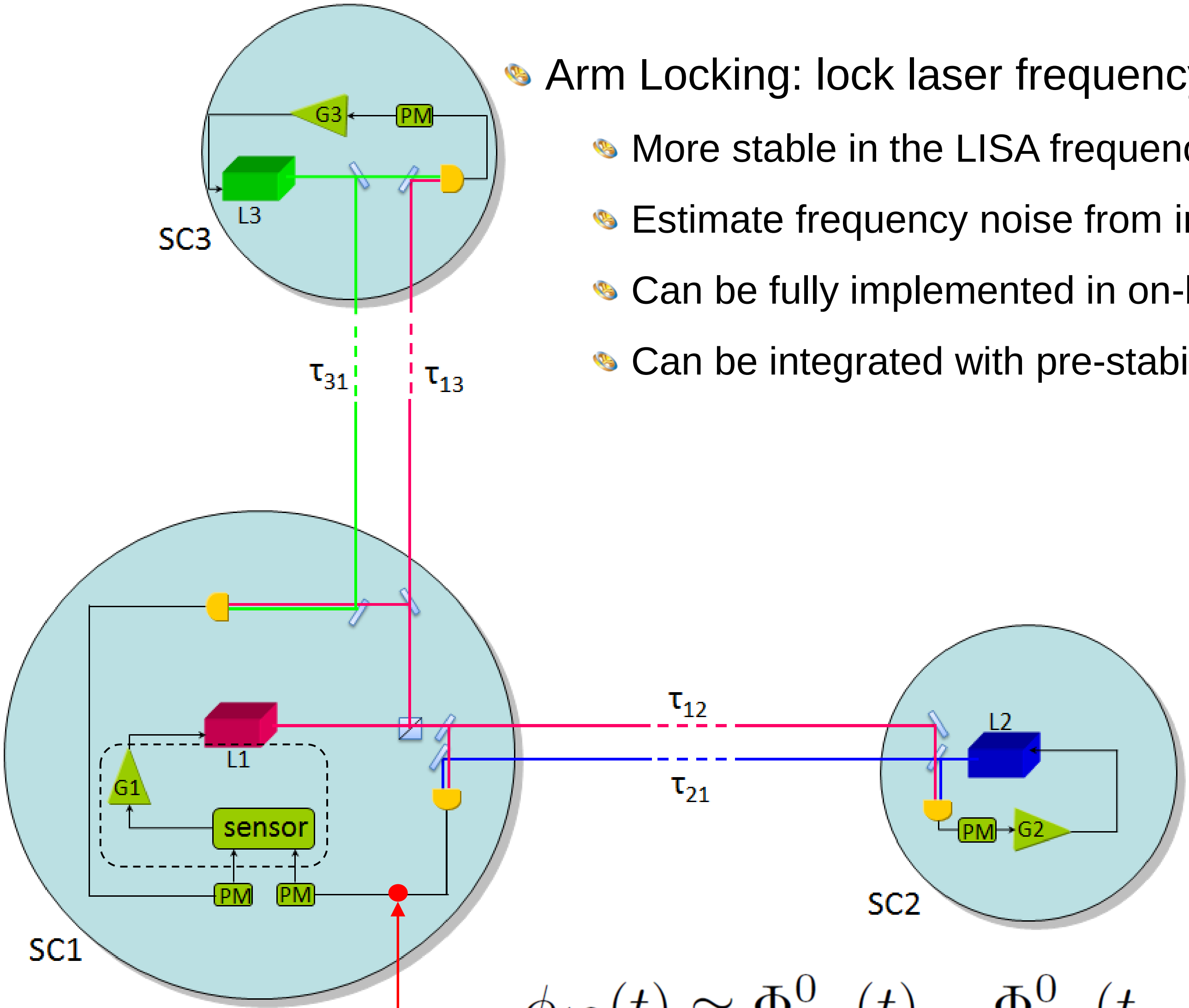


- Lock laser to a local frequency reference
  - Fabry-Perot cavity
  - Mach-Zehnder
  - Molecular line

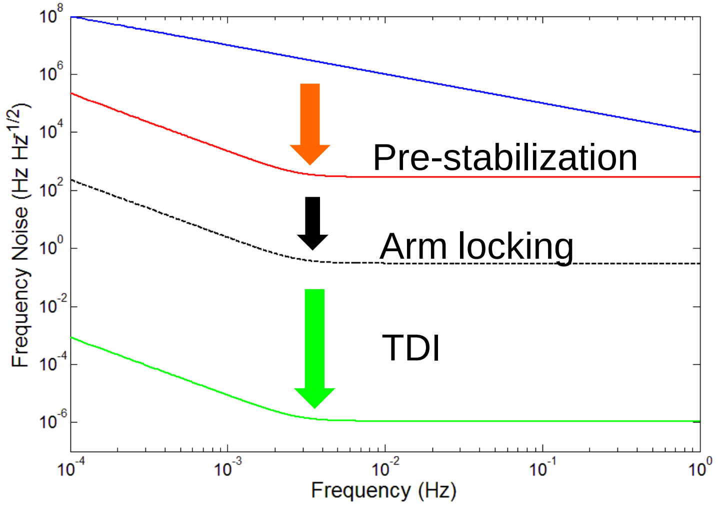


Arm Length as a Better Reference

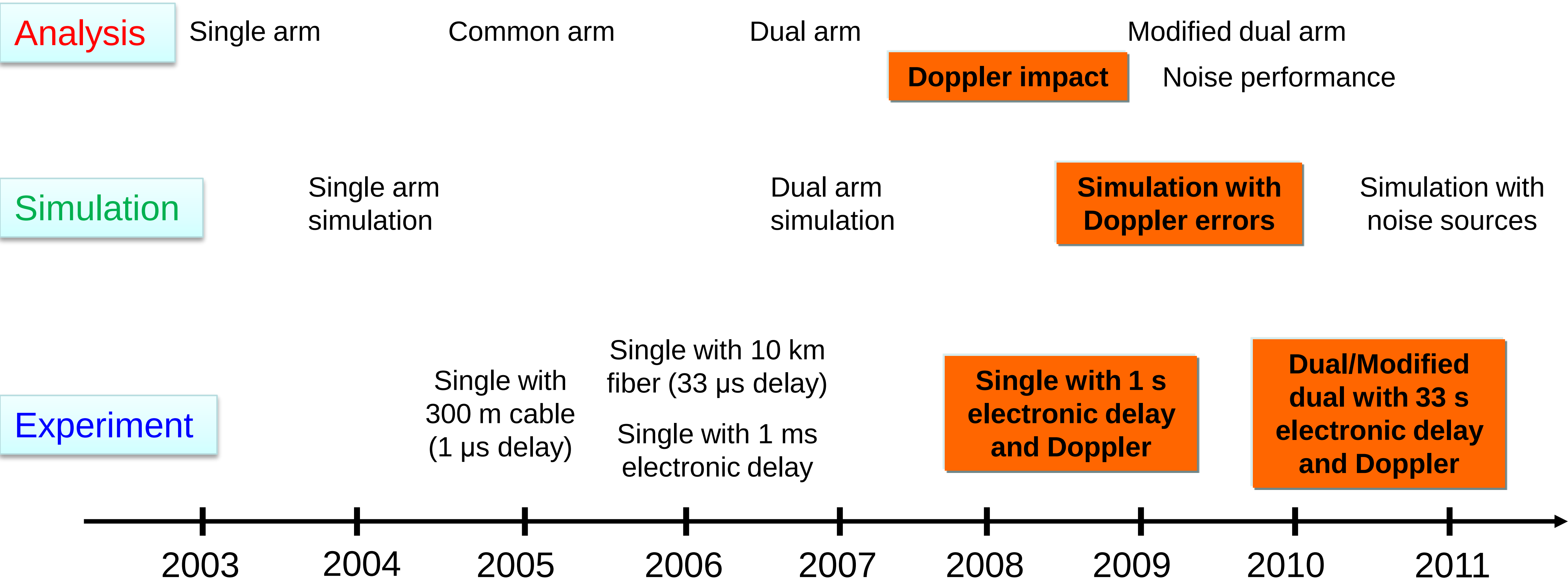
- Arm Locking: lock laser frequency to the LISA arms
  - More stable in the LISA frequency band: $\delta L/L \sim 10^{-21}\ \mathrm{Hz}^{-1/2}$
  - Estimate frequency noise from inter-SC phase measurements
  - Can be fully implemented in on-board data processing
  - Can be integrated with pre-stabilization system (requires tunable reference)



$$\phi_{12}(t) \approx \Phi^0_{L1}(t) - \Phi^0_{L1}(t-\tau_2)$$



Arm Locking Chronicle



Analysis

Single arm

Common arm

Dual arm

Modified dual arm

Doppler impact

Noise performance

Simulation

Single arm simulation

Dual arm simulation

Simulation with Doppler errors

Simulation with noise sources

Experiment

Single with 300 m cable (1 μs delay)

Single with 10 km fiber (33 μs delay)

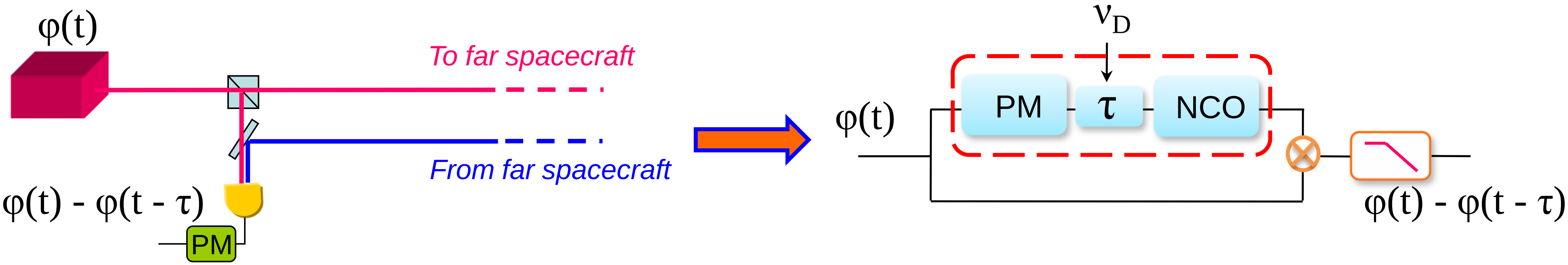
Single with 1 ms electronic delay

Single with 1 s electronic delay and Doppler

Dual/Modified dual with 33 s electronic delay and Doppler

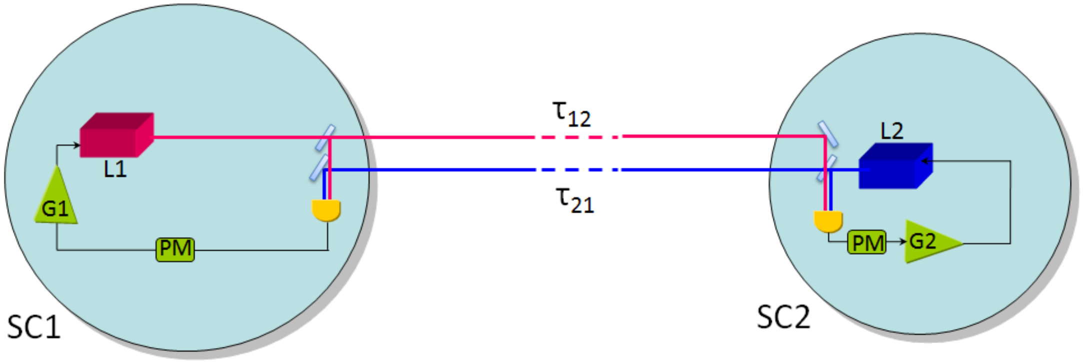
Arm Locking in Lab

How to reproduce the 5 Gm arm length in lab?



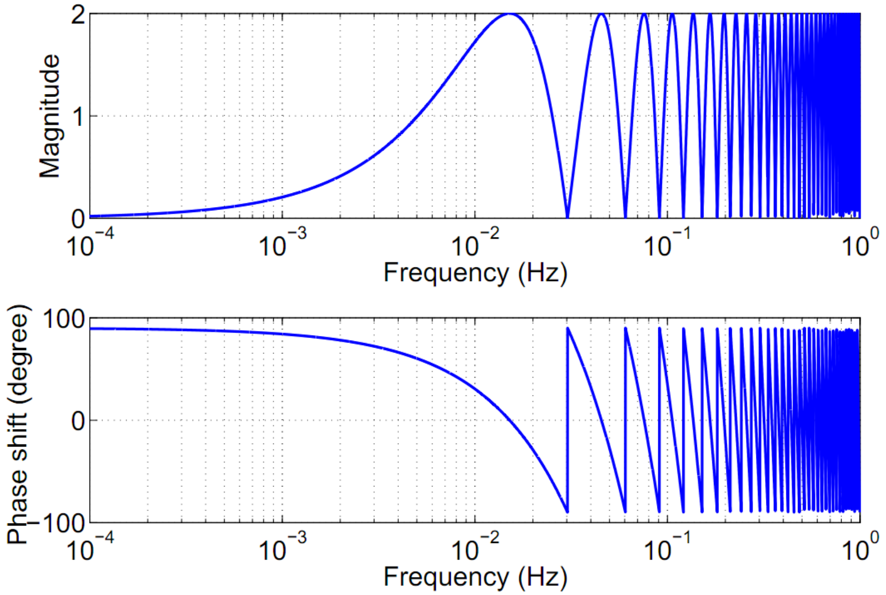
LISA	Interferometer Simulator
1064 nm laser	1064 nm laser MHz beat
5 Gm long arm length	16 s electronic phase delay (EPD)
Doppler shifts	MHz frequency shifts in EPD
Constellation phase locking	Analog PLL
Heterodyne interference	Analog mixing
Phasemeter	DPLL phasemeter prototype
Arm locking sensing & control law	Sensor & controller on FPGA
Integration with pre-stabilized lasers	Heterodyne PLL / PZT cavity

Single Arm Locking



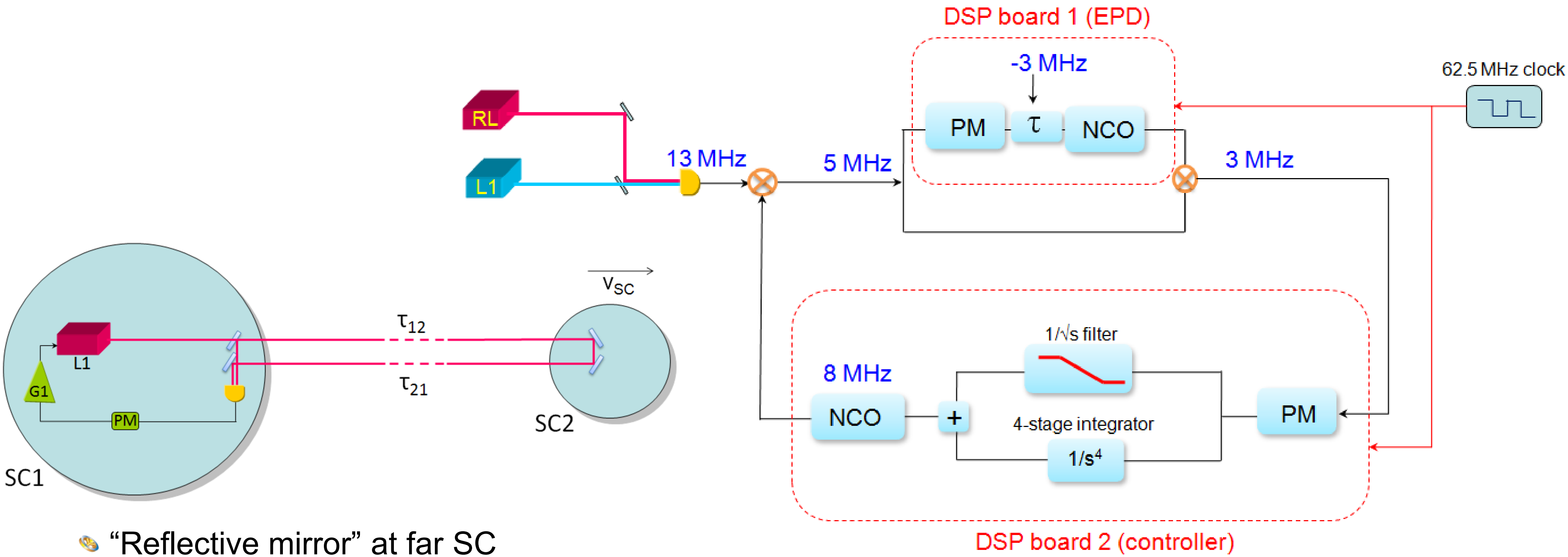
- Mach-Zehnder interferometer with 10 Gm arm length mismatch
- Interferometer response:

$$P_{1i}(s) = \frac{\phi_{1i}(s)}{\Phi^0_{L1}(s)} \approx 1 - e^{-s\tau_i}, \qquad i = 2,3$$



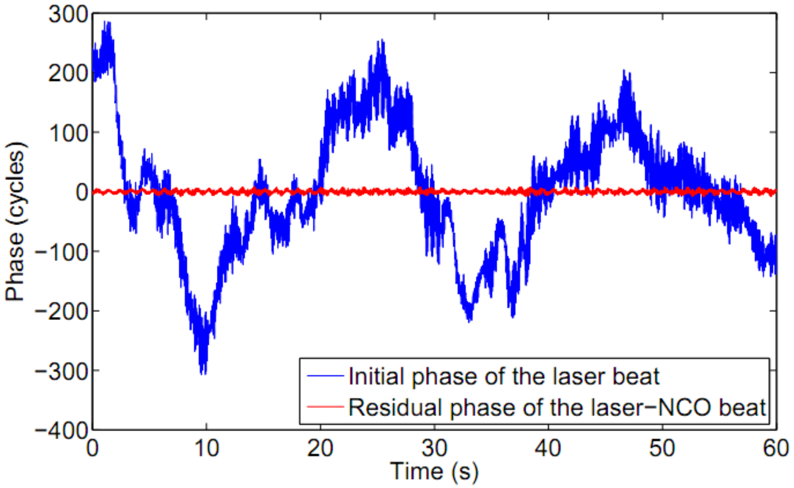
- insensitive at interferometer nulls
- bandwidth is not limited by 33 s time delay
- controller design needs to carefully keep enough phase margin
- control system is verified in simulations and experiments (with very short delay times)

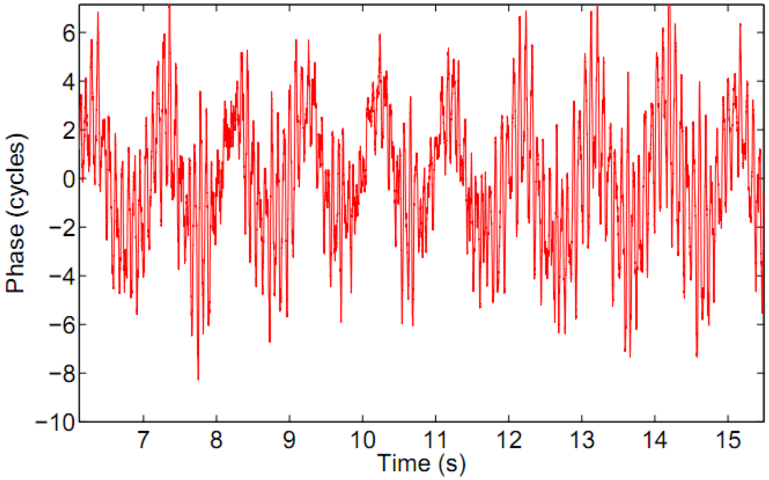
Control System - Single

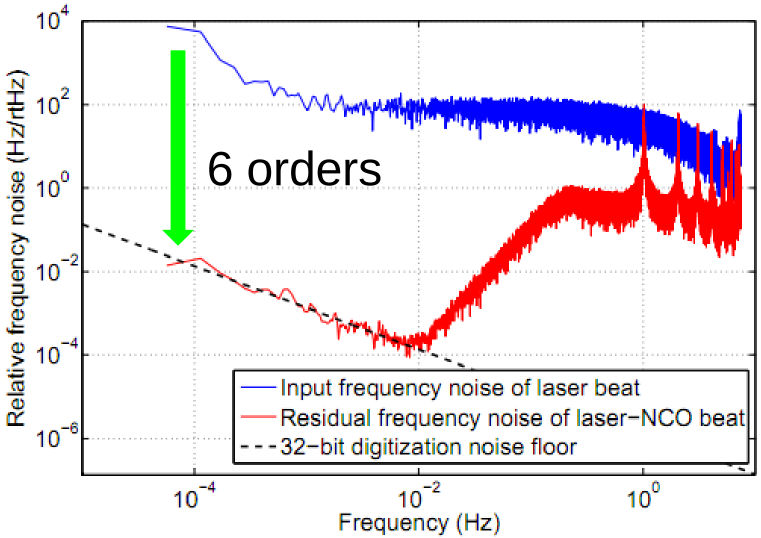


- “Reflective mirror” at far SC
- NCO tracks the input phase noise
- 1 s delay, Doppler shift
- common clock for both DSP boards

Control System - Single





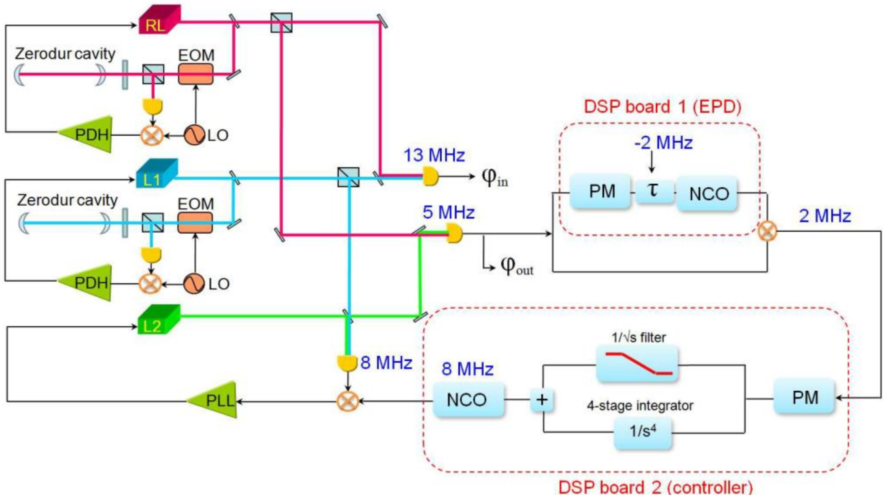


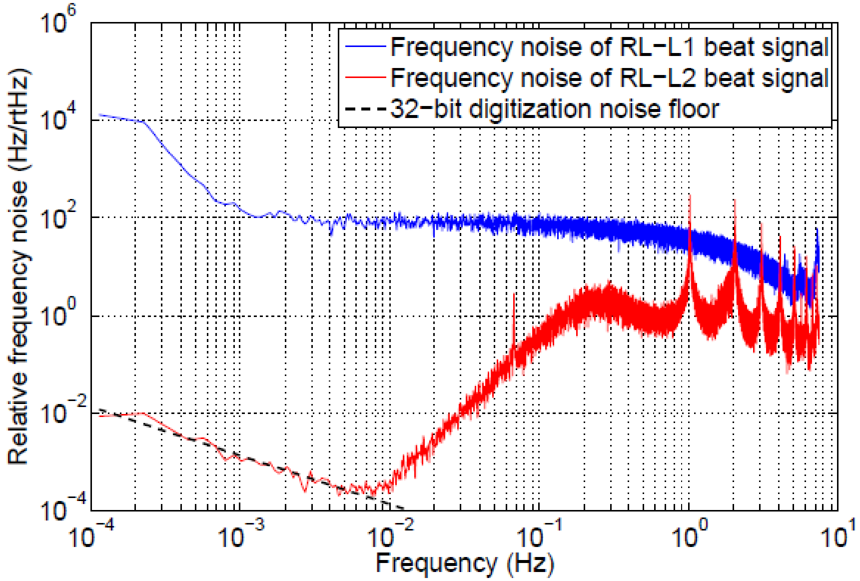
- First peak at 1 Hz
- 6 orders of magnitude suppression
- Limited by 32-bit digitization noise

In realistic case, a 33 s delay would generate the first peak at 30 mHz, well inside the LISA band.

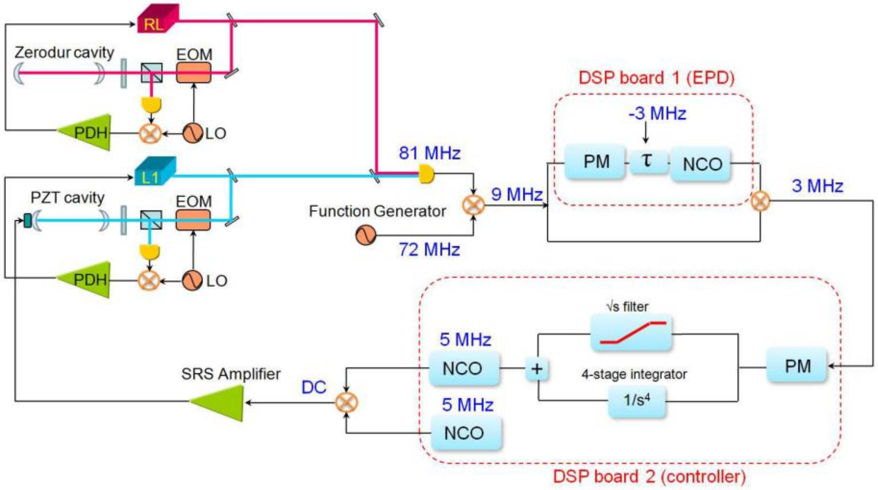
Single AL with Pre-stabilized Laser

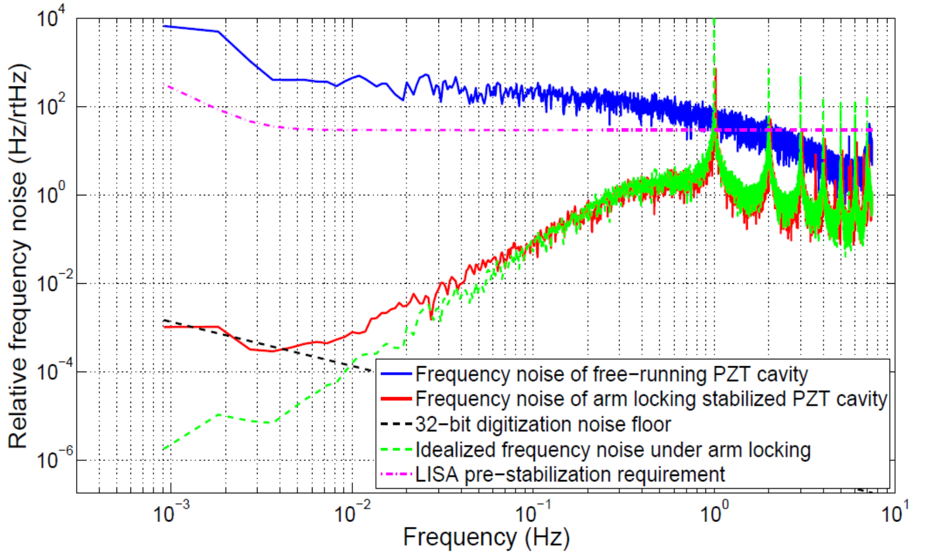
Heterodyne PLL





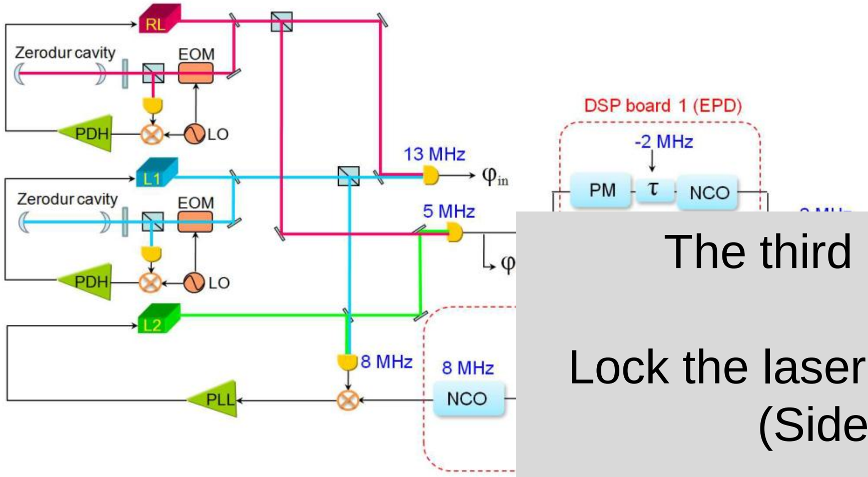
PZT actuated cavity

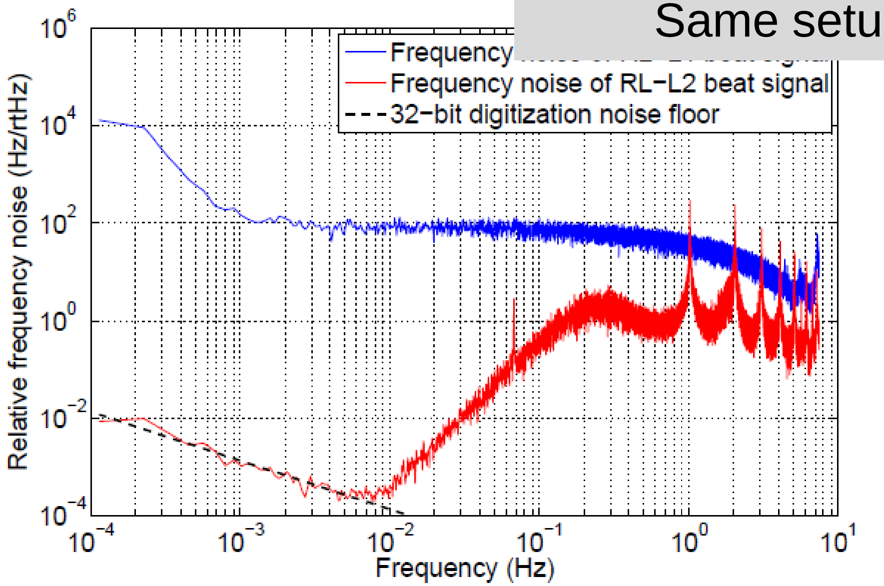




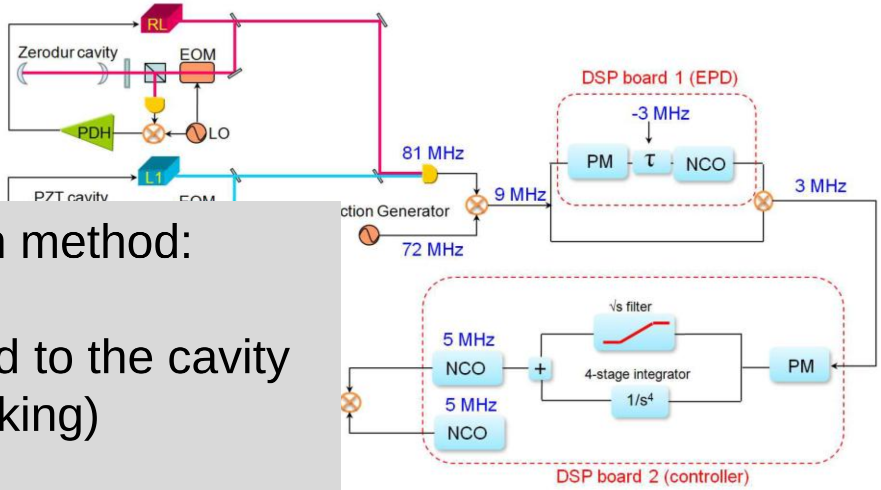
Single AL with Pre-stabilized Laser

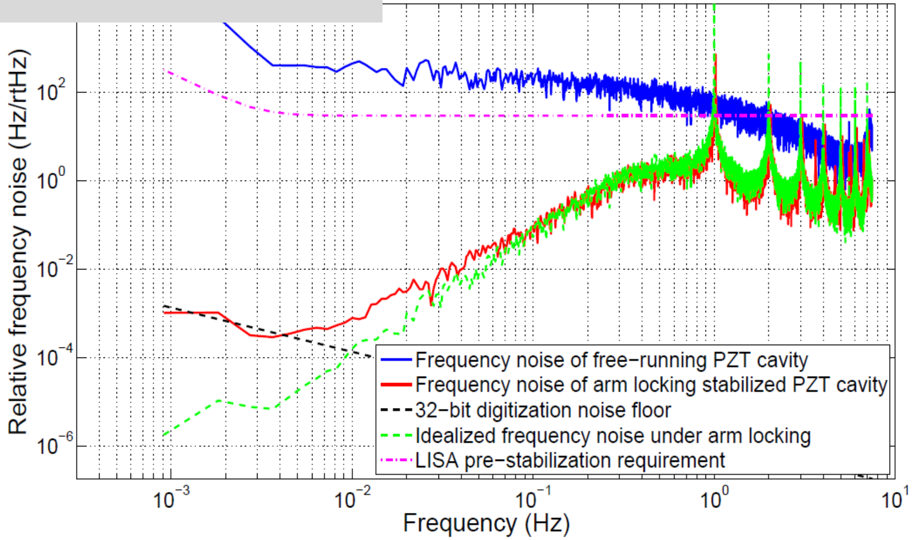
Heterodyne PLL





PZT actuated cavity





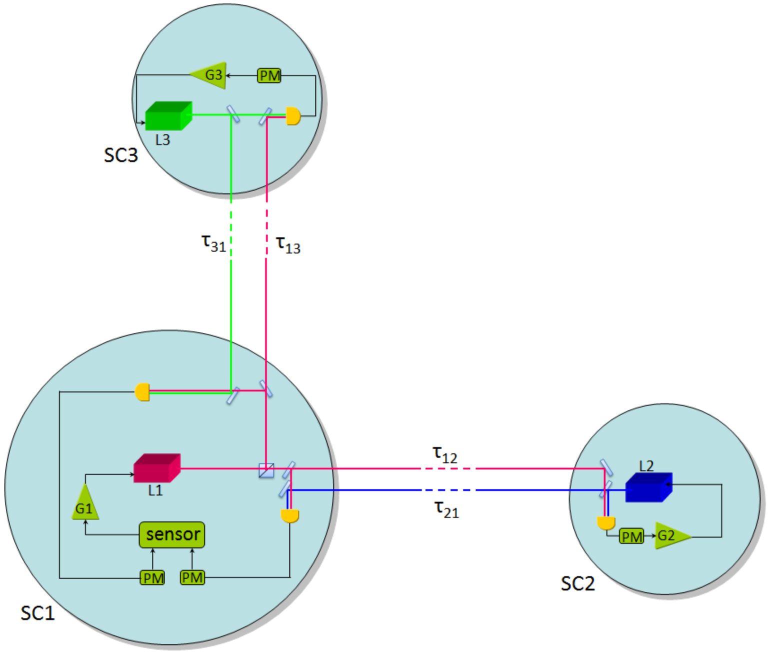
The third actuation method:

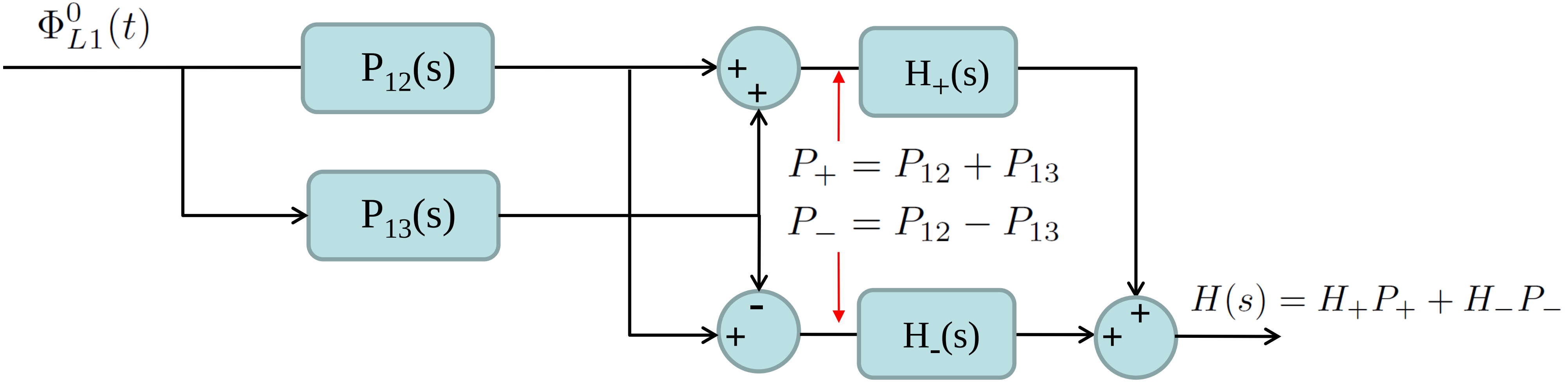
Lock the laser sideband to the cavity
(Sideband locking)

Same setup tested at GSFC

Dual Arm Sensor

- push sensor nulls beyond the LISA band
- better bandwidth performance
- better noise suppression performance
- never verified in experiments before



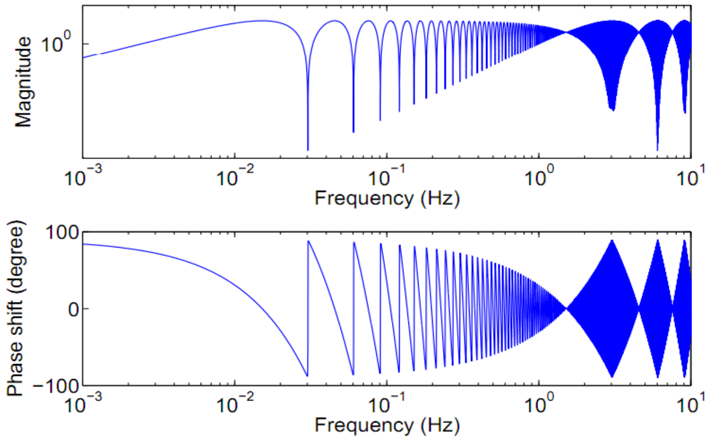
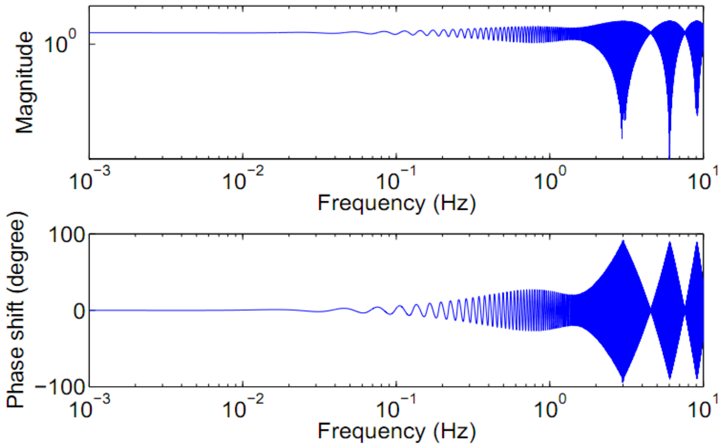


$$P_+ = P_{12} + P_{13}$$

$$P_- = P_{12} - P_{13}$$

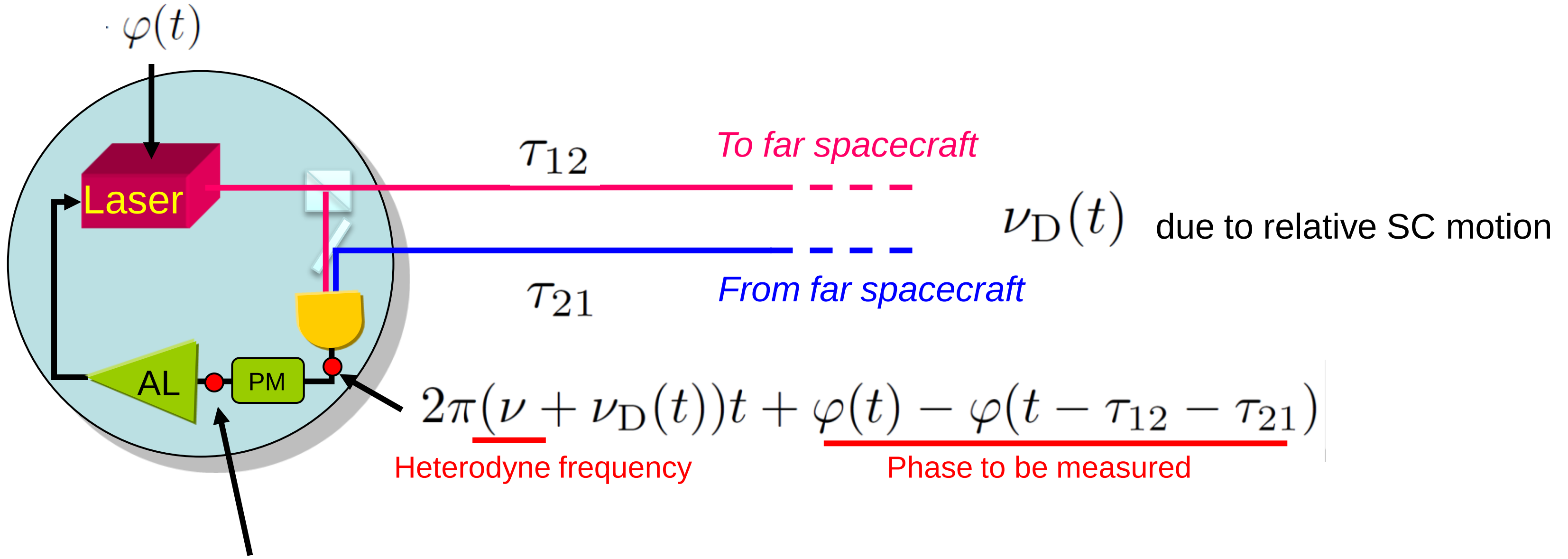
$$H(s) = H_+P_+ + H_-P_-$$

Common/Dual Arm Locking

Sensor	Sensor	Transfer Function
Common	$H_+(s) = 1,$ $H_-(s) = 0$	
Dual	$H_+(s) = 1,$ $H_-(s) = \frac{E(s)}{s\Delta\tau}$ E (s) – low-pass filter @ $1/4\Delta\tau$	

Frequency Pulling

- Doppler shifts from relative SC motions (±18 m/s → ±17 MHz)
- cause a Doppler error in phase measurements



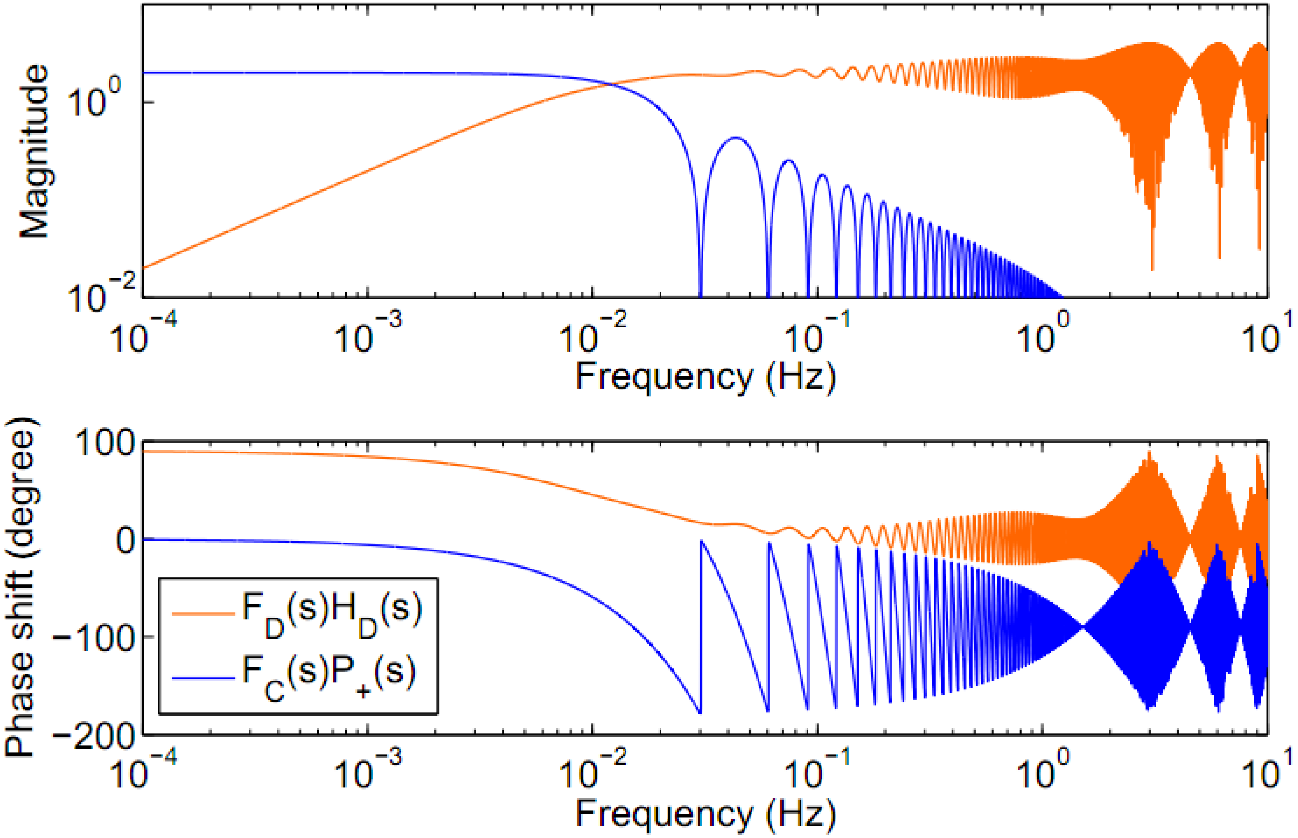
$$2\pi(\underbrace{\nu}_{\text{Heterodyne frequency}} + \nu_{\mathrm{D}}(t))t + \underbrace{\varphi(t) - \varphi(t - \tau_{12} - \tau_{21})}_{\text{Phase to be measured}}$$

$$2\pi \underbrace{\Delta\nu_{\mathrm{D}}(t)}_{\text{Doppler frequency error}} t + \varphi(t) - \varphi(t - \tau_{12} - \tau_{21})$$

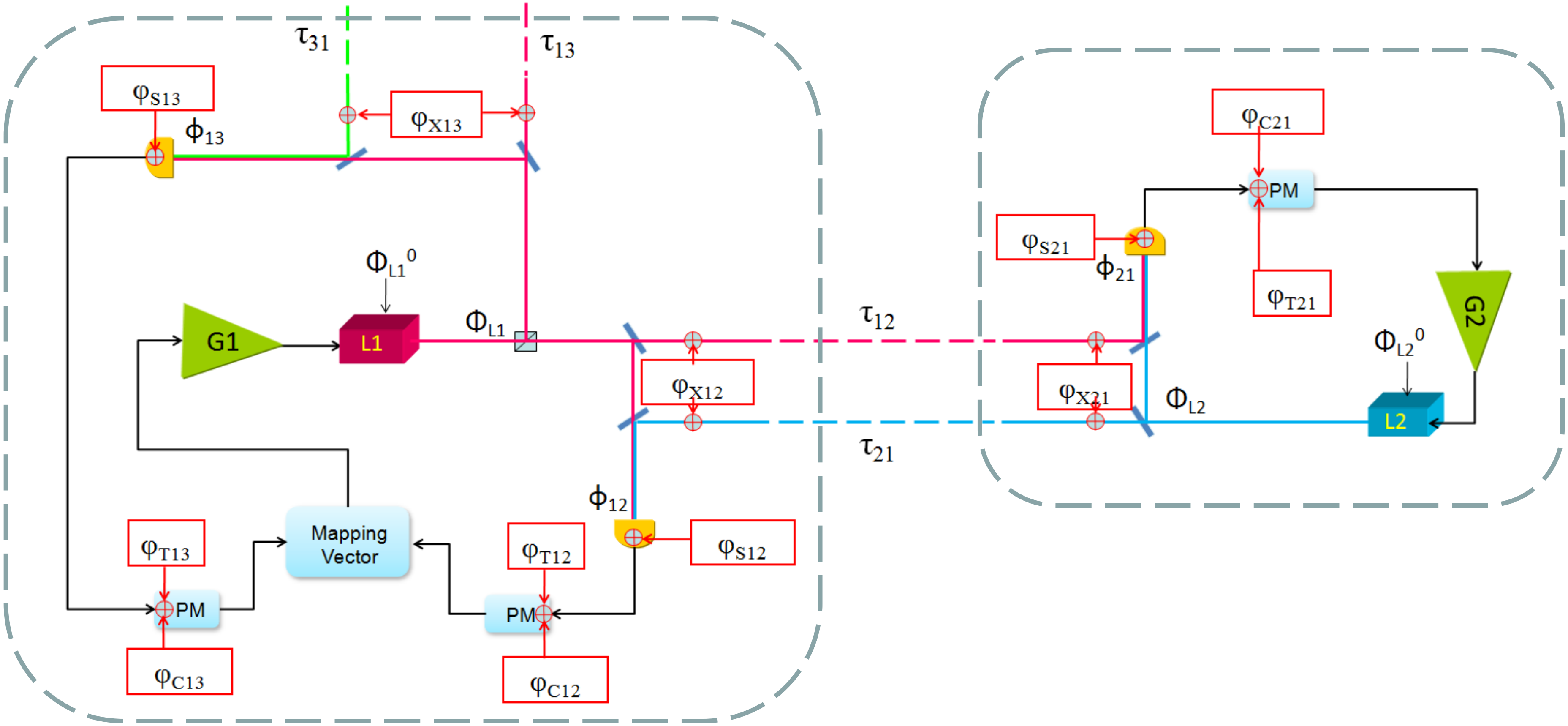
	Frequency pulling rate
Single	$\Delta\nu_{\mathrm{D}}(t)/\tau$
Common	$\Delta\nu_{\mathrm{D}+}(t)/2\bar{\tau}$
Dual	$\Delta\nu_{\mathrm{D}-}(t)/2\Delta\tau$

Modified Dual Arm Locking

- A combination of
  - common arm sensor at f < 30 mHz (frequency pulling $\Delta\nu_{\mathrm{D}+}(t)/2\bar{\tau}$ & noise advantage)
  - dual arm sensor at f > 30 mHz (gain advantage)

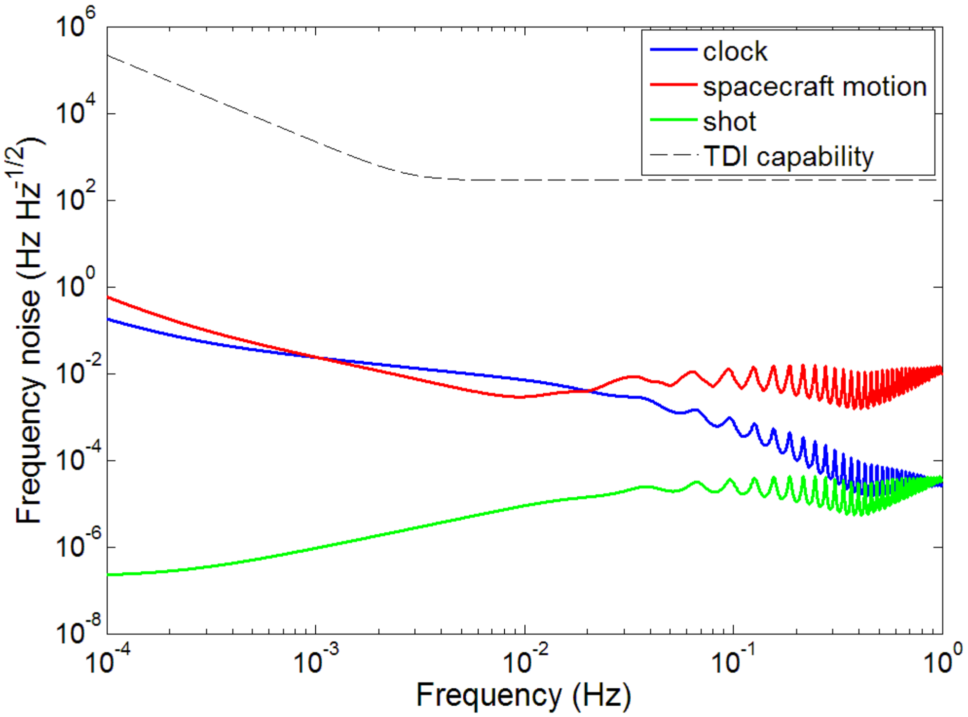


Noise Model of Arm Locking



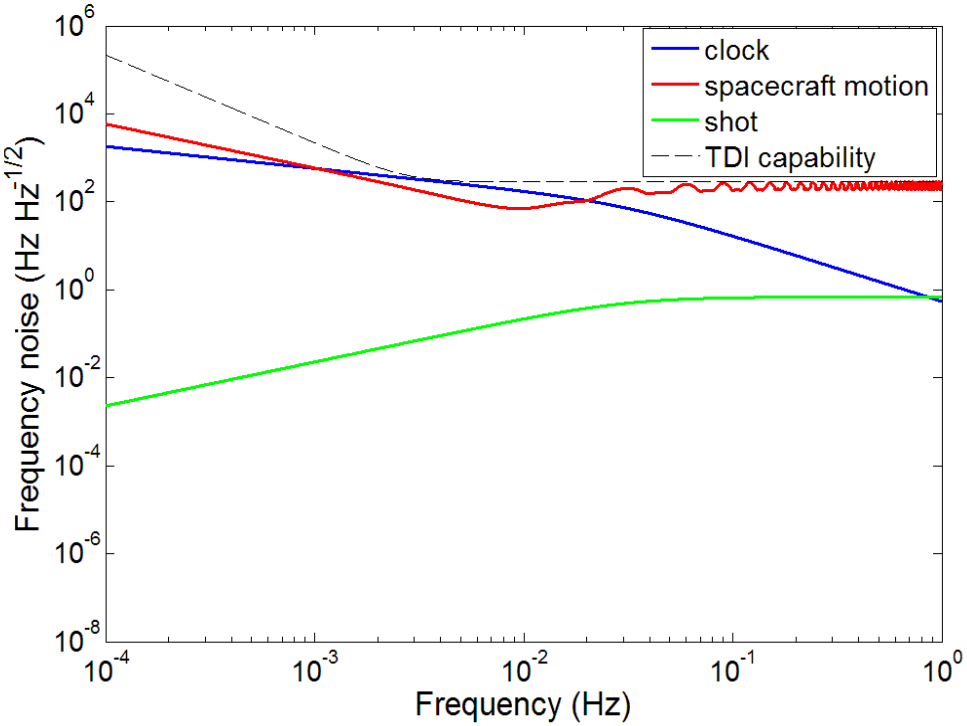
- clock noise – phase noise of the referencing clock in phase measurements
- SC motion – arm length uncertainty from the DRS
- shot noise – limited number of photons received per second by photodiodes
- technical noise – ADC noise, digitization noise, etc. in signal processing

Noise Floors of MDAL



Maximum arm length mismatch

$2\Delta\tau = 0.51$ s, $\Delta L = 76{,}000$ km



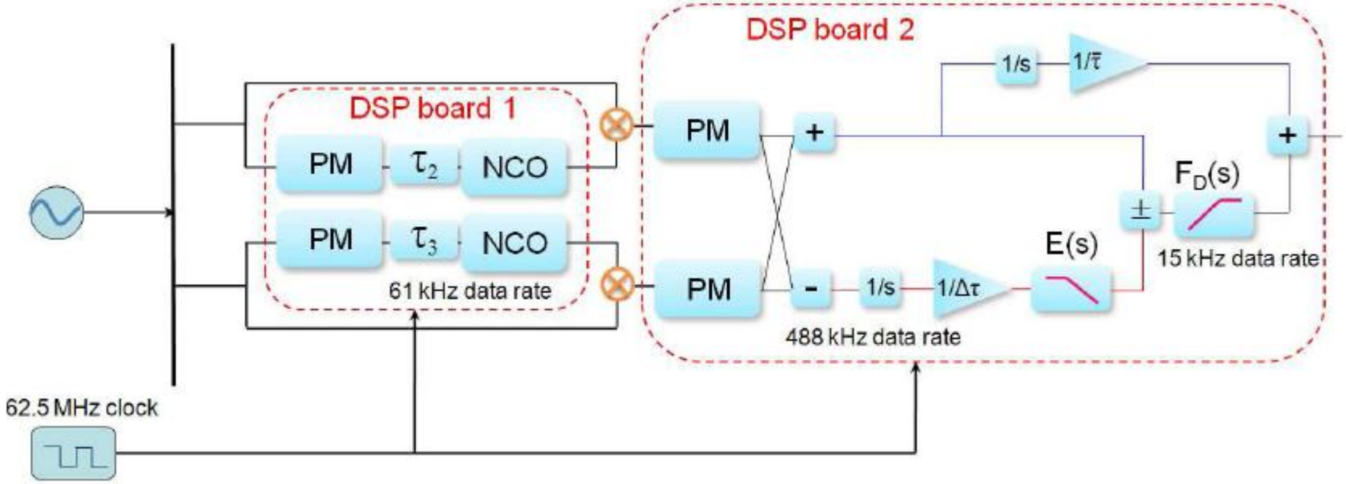
Minimum arm length mismatch
that meets the requirement

$2\Delta\tau = 20\ \mu$s, $\Delta L = 3$ km

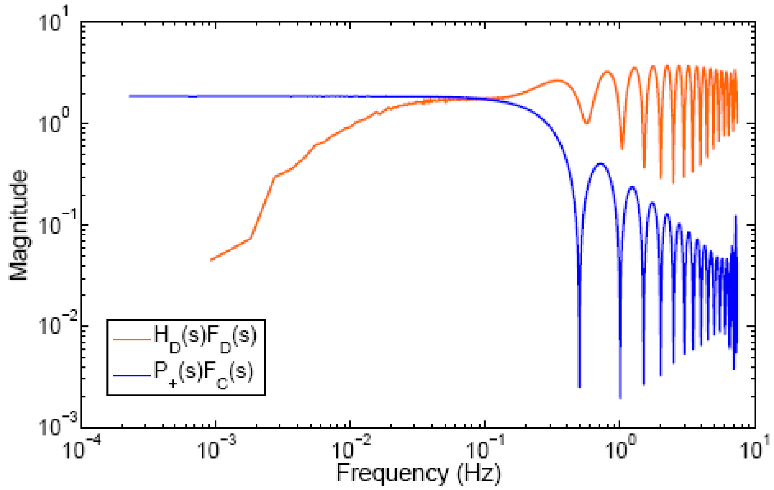
Worse scenario only lasts for ~ 300 s, twice a year.

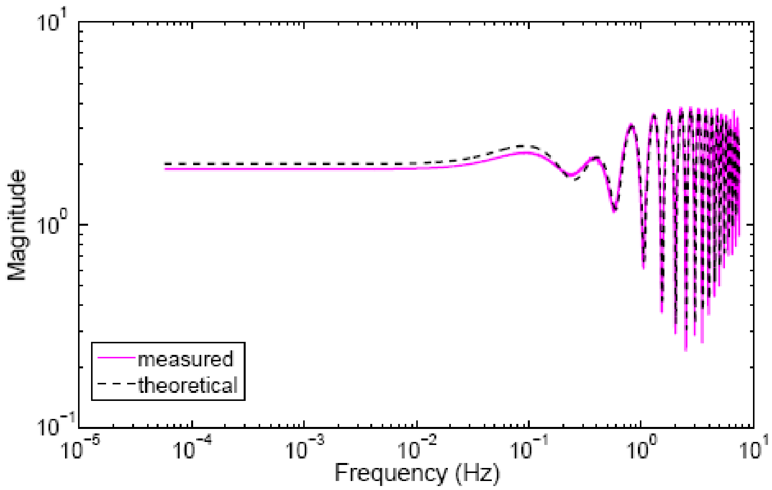
Arm locking alone can meet the pre-TDI requirement (with all six links operational).

Control System – Modified Dual

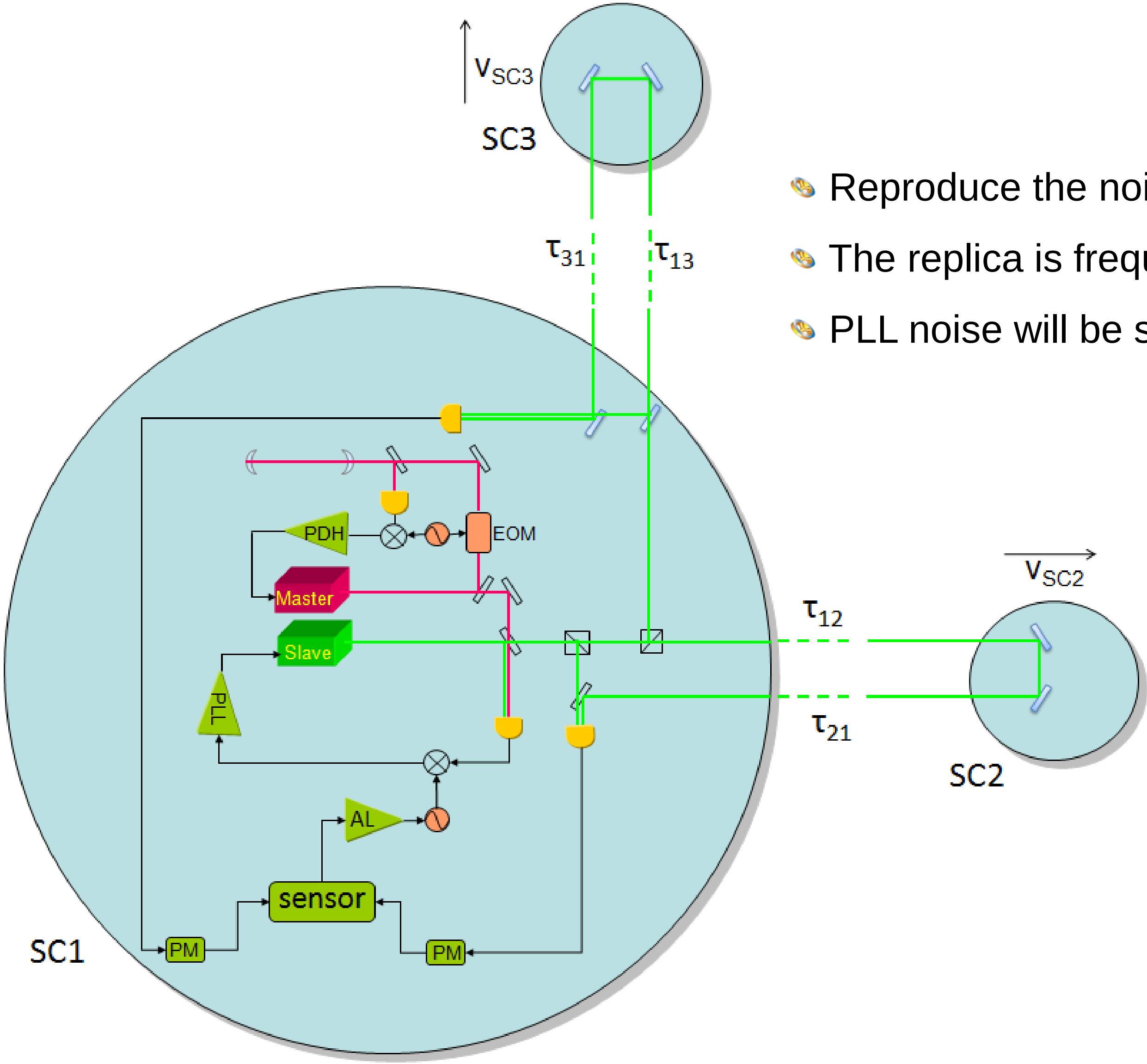


- High-pass filter the dual arm
- Integrate the common arm



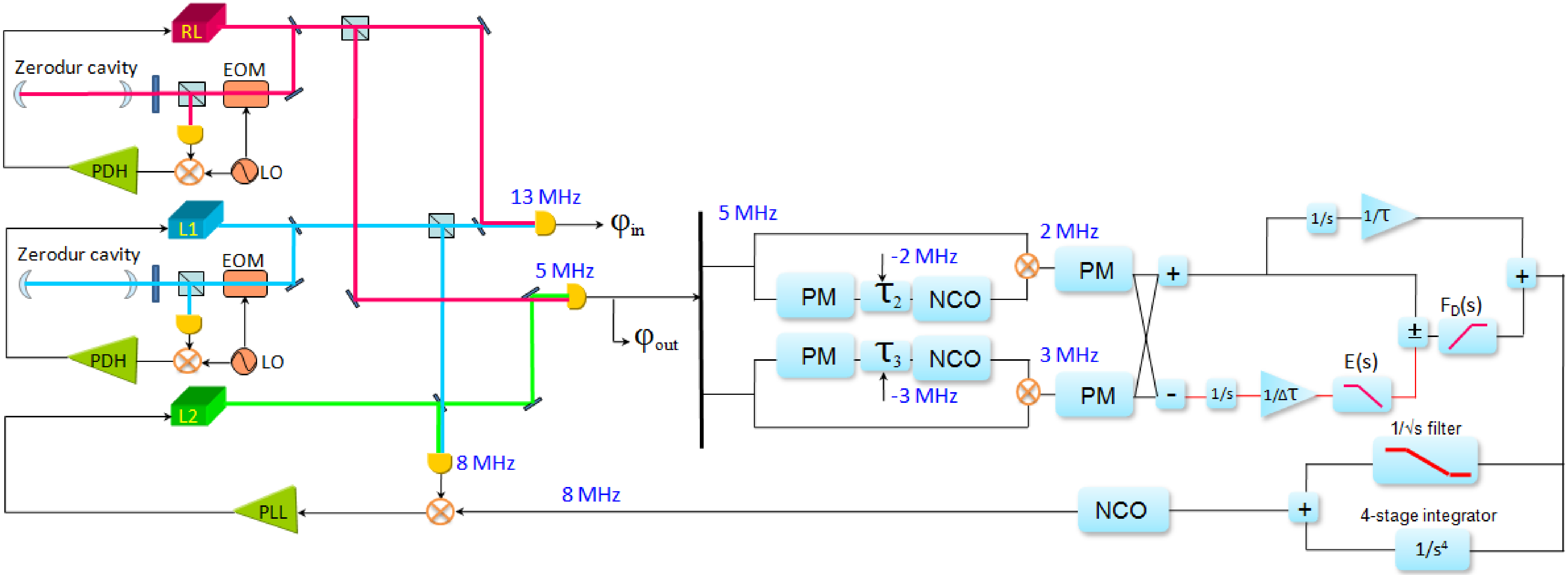


MDAL with Pre-stabilized Laser



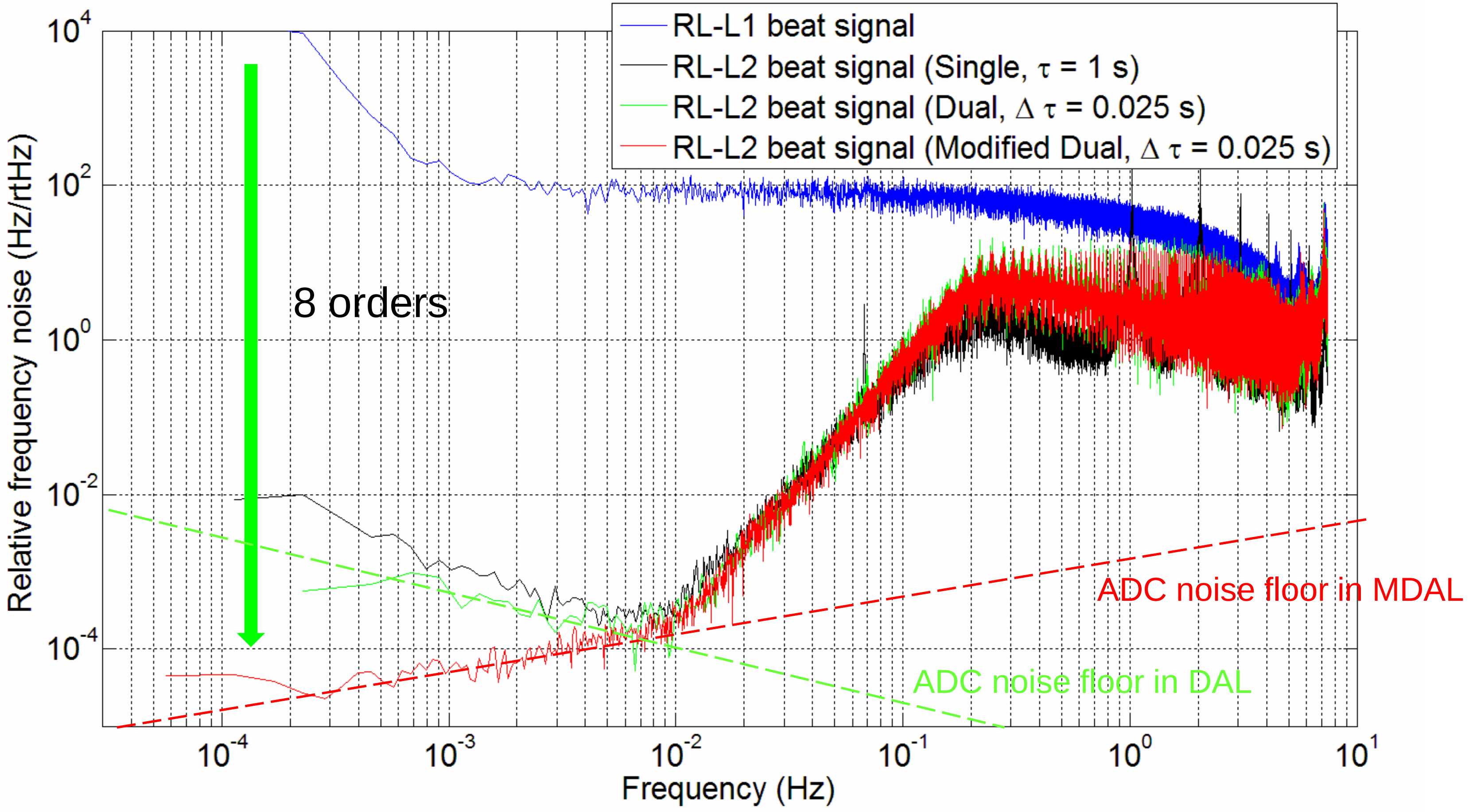
- Reproduce the noise property via phase locking
- The replica is frequency tunable
- PLL noise will be suppressed by the arm locking gain.

MDAL with Pre-stabilized Laser



- 48-bit sensor & controller
- $\bar{\tau} = 33\ \mathrm{s}, \Delta\tau = 0.025\ \mathrm{s}$
- Tested with dual & modified dual arm locking

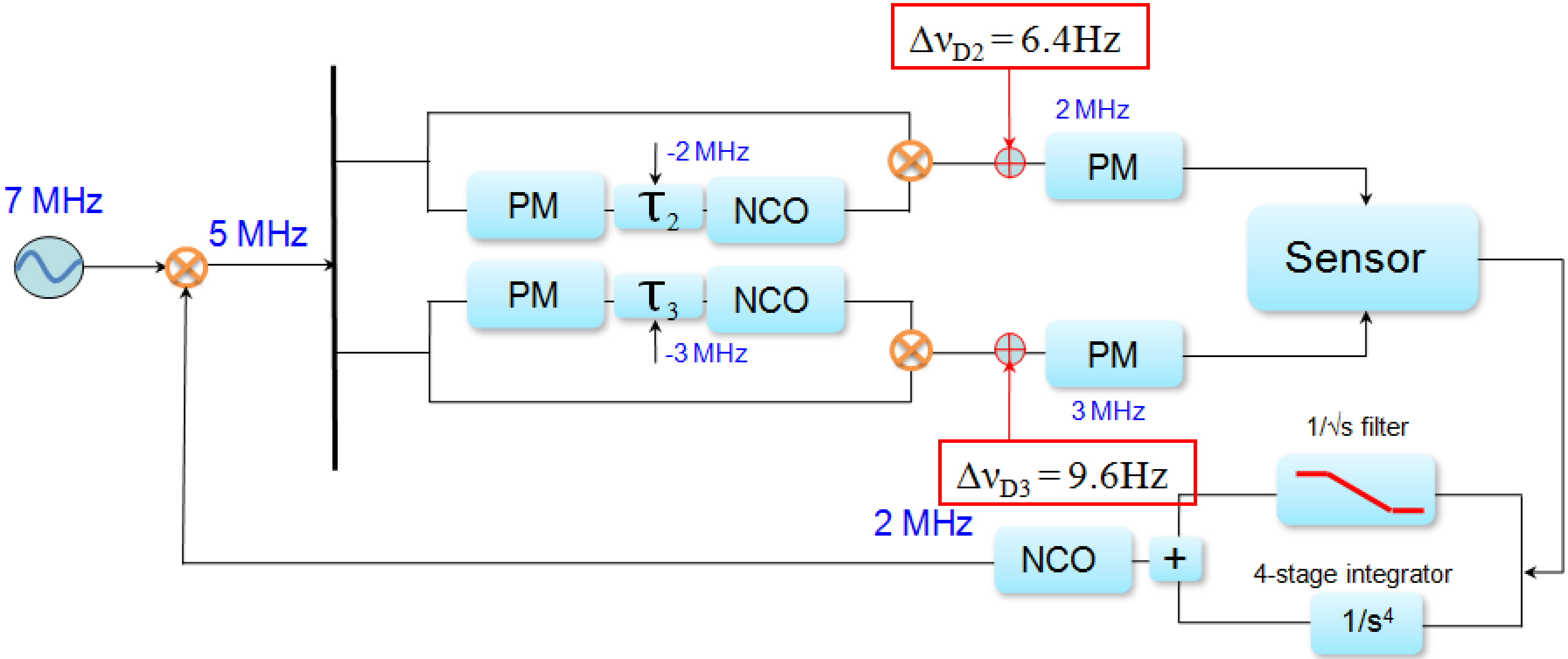
Noise Performance



Noise limited region

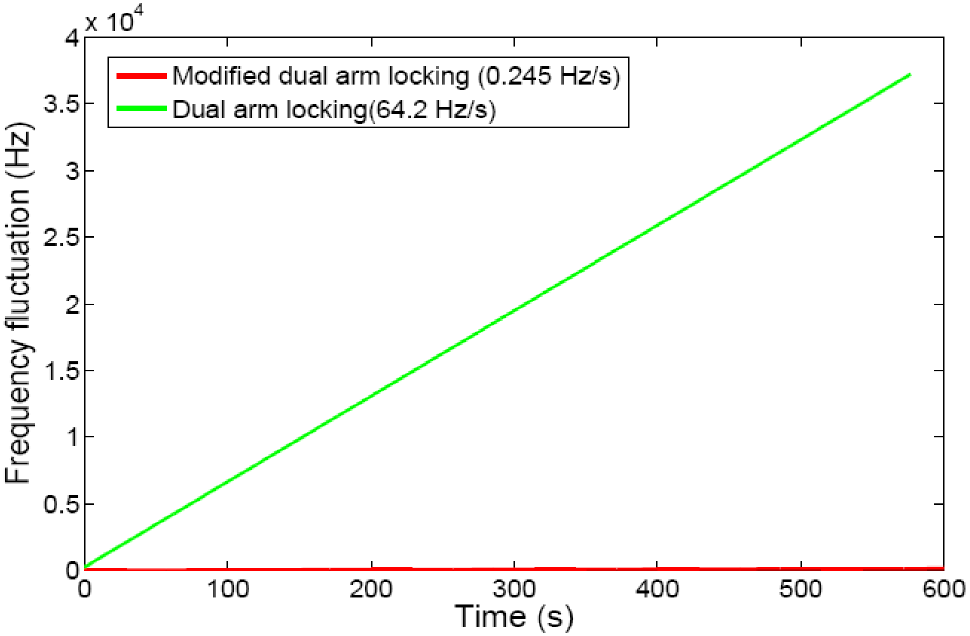
Gain limited region

Arm Locking with Doppler Errors



- generate Doppler frequency error by shifting the clock frequency
- $\bar{\tau} = 33\ \mathrm{s}, \Delta\tau = 0.025\ \mathrm{s}$
- tested with dual & modified dual arm locking

Drift in Steady State

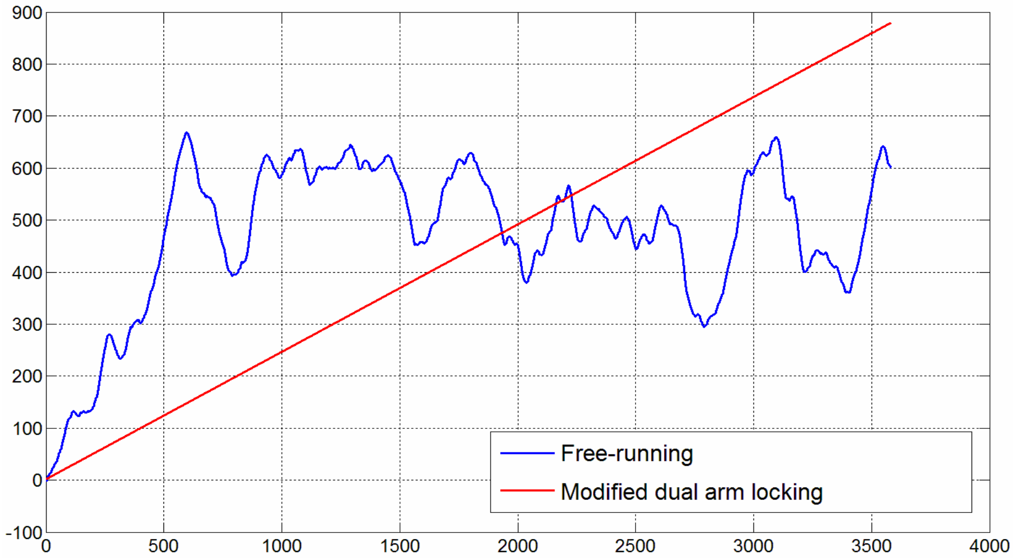


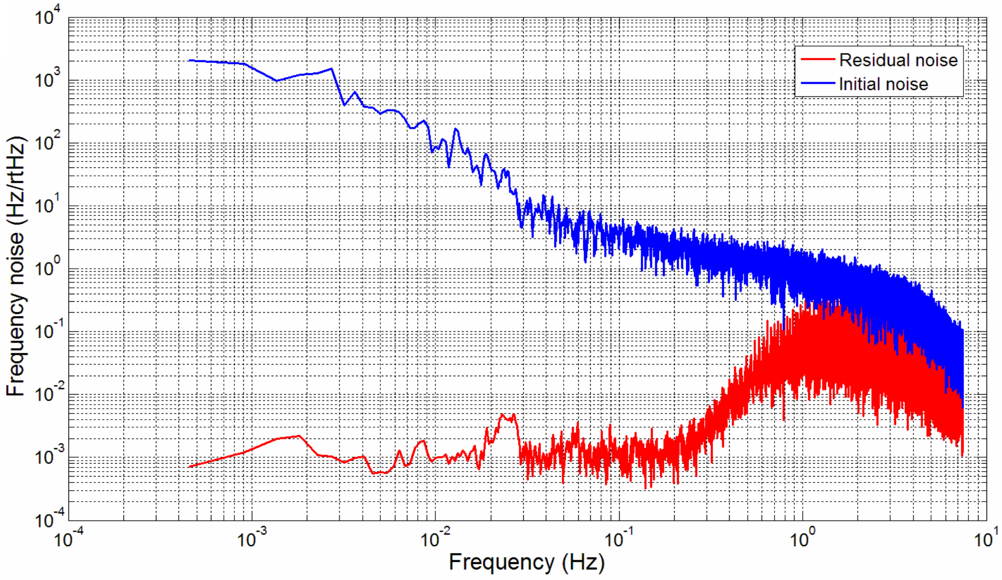
- Drift rate in dual arm locking:

$$\frac{\Delta\omega_{D-}}{2\Delta\tau} = 64\ \mathrm{Hz/s}$$

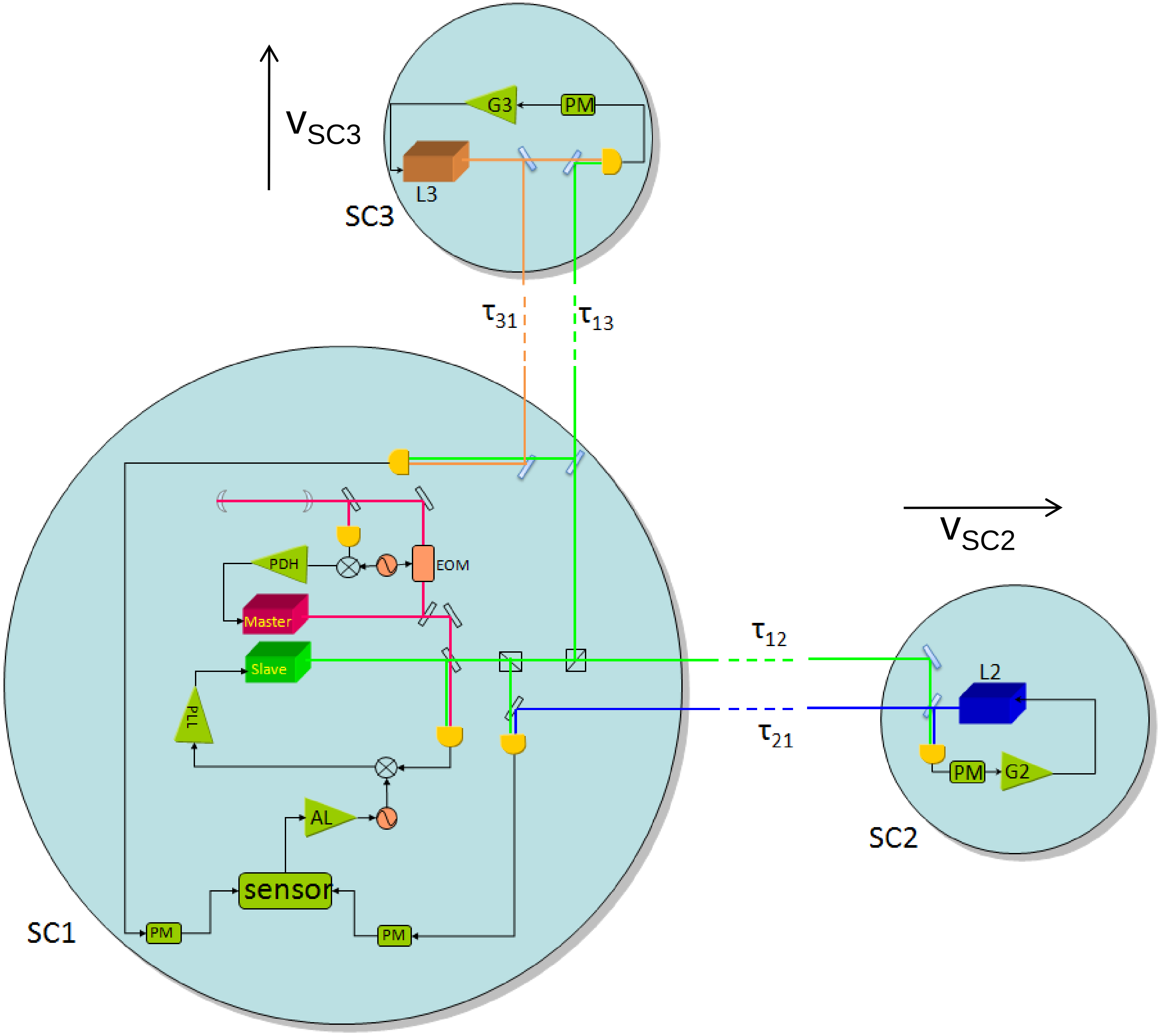
- Drift rate in modified dual arm locking:

$$\frac{\Delta\omega_{D+}}{2\bar{\tau}} = 0.24\ \mathrm{Hz/s}$$

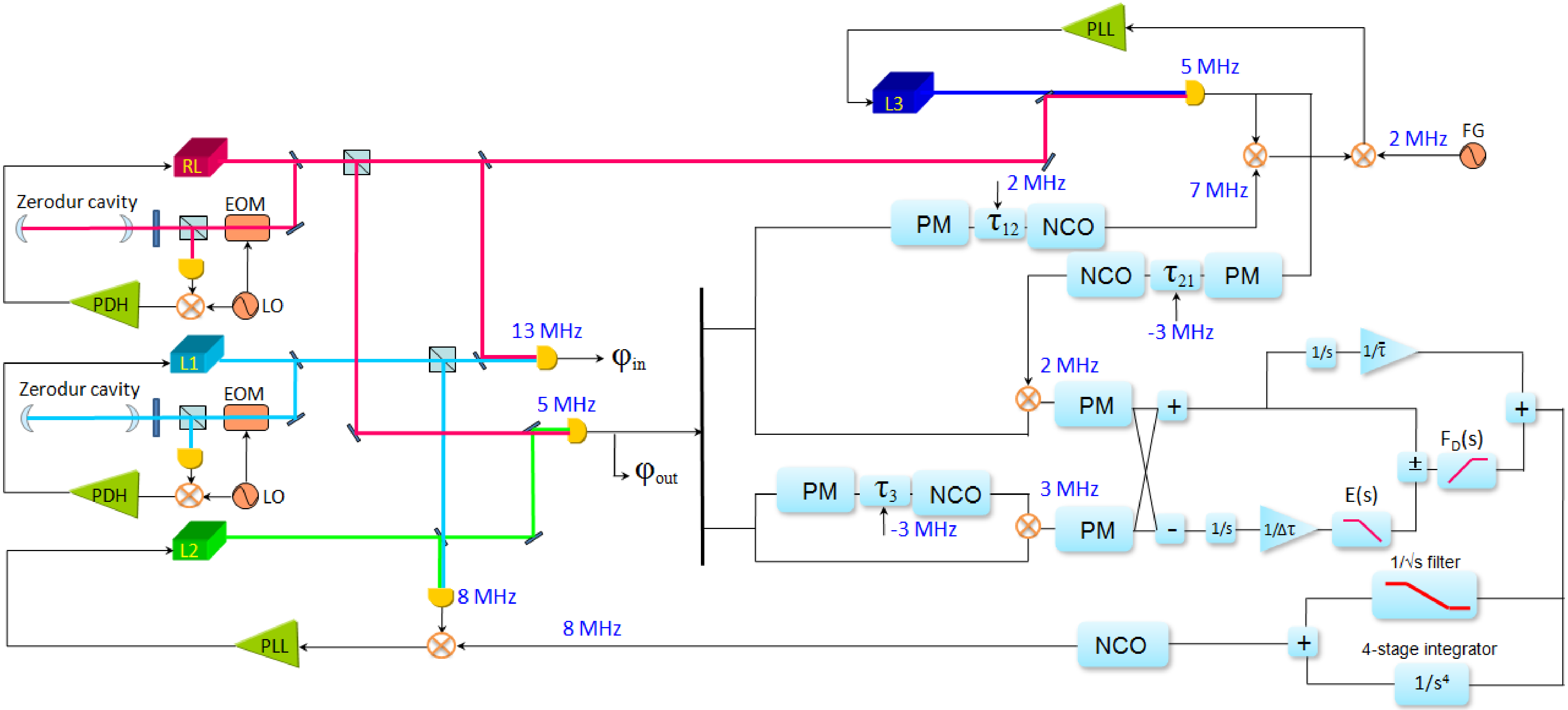




Put Far-end PLLs Back

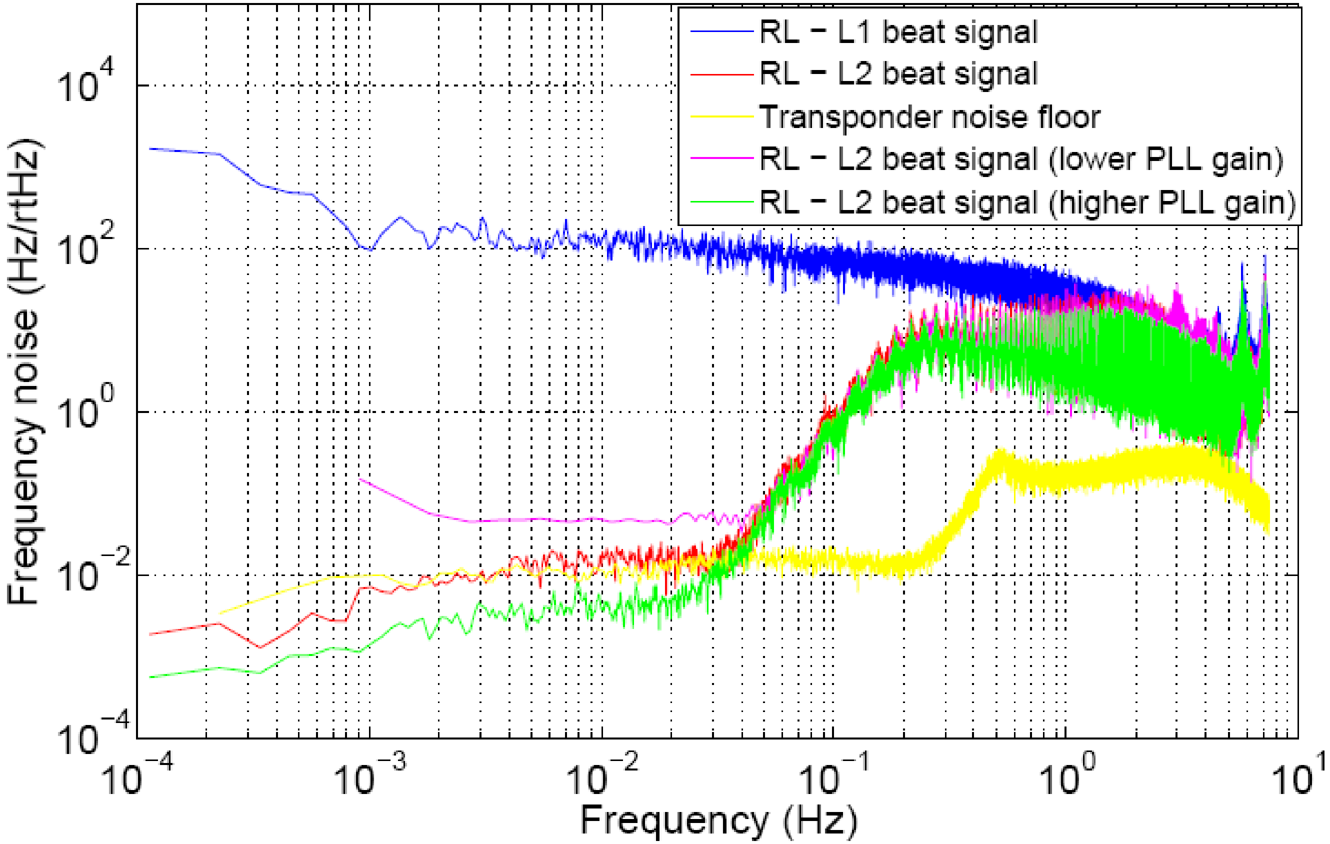


Integrated with Far-end PLL



- Phase-lock the far-end laser to the delayed signal
- Function generator is synchronized to the master clock
- Transponder noise includes limited PLL gain and clock noise

Results



- Still sufficiently meet the requirement with a margin of 25,000 at 3 mHz.
- The stabilized frequency noise is consistent with the transponder noise floor.
- The noise floor can be adjusted by changing the PLL gain.

Conclusion

- Control system of arm locking
  - Verified different arm locking schemes under realistic long time delay and Doppler shifts
  - Integrate with cavity stabilized lasers and sufficiently meet the LISA requirement
- Noise limitations of arm locking
  - Dominated by digitization noise or ADC noise in our experiments
  - Demonstrate the noise advantage of modified dual arm locking
  - Demonstrate arm locking performance in the presence of non-negligible transponder noise
- Doppler frequency error in arm locking
  - Studied the frequency pulling in different arm locking schemes

Arm locking has solved the problem of laser frequency stabilization for LISA and can be used in the frequency stabilization for future similar space-based interferometric detectors.

Current Status of ESA-led LISA…

- As announced by ESA in March 2011, due to a modified international cooperation scenario, it is now necessary to study a European-only mission that offers a significant reduction of the cost while maintaining its core science objectives.

- Whilst maximizing the use of results of the LISA studies performed so far and the heritage from LISA Pathfinder, a number of significant changes to the payload, the spacecraft and the mission architecture have to be made to enable the mission to fit the new budget profile.

- The goal is to be ready for a technical and programmatic review starting in November 2011 and for a scientific review by the ESA advisory bodies thereafter, in order to support an SPC downselection for entering phase A/B1 in February 2012.

http://sci.esa.int/science-e/www/object/index.cfm?fobjectid=48728